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The Dynamics of Objects in Orbits Resembling That of P/Encke

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The overall orbital evolution of bodies resembling that of P/Encke is studied by numerical integration. Dynamical paths are found that connect orbits of this type to their possible sources, i.e., the asteroidal main belt and the Jupiter family of comets. Possible end-states for these orbits include ejection from the region of the inner planets, due to close encounters with Jupiter, and collision with the Sun. We find a purely gravitational dynamical path (i.e., one not involving nongravitational forces) connecting the orbit of P/Encke to the orbits of the rest of the Jupiter family of comets on a timescale of several times 10^5 years, somewhat longer than typical cometary physical lifetimes. The hypothesis of a genetic relationship among some or all of the bodies currently in Encke-like orbits is not supported by our results. The existence of dynamical channels linking these orbits to asteroidal and cometary sources explains why both types of objects are present in the Taurid complex and points to the presence of both high-density and low-density material in the related meteoroid population.

1. INTRODUCTION

Among the members of the Jupiter family of comets P/Encke is a rather exceptional object. It is the one with the largest number of recorded apparitions and is observable throughout its orbit; moreover, it is usually assumed to be the parent comet of the Taurid meteoroid complex, a broad, evolved complex of meteor streams that contains a sizable fraction of the meteoroid population of the Earth's space environment (Whipple 1967, Kresák and Kresáková 1987, Porubčan and Štohl 1987, Štohl and Porubčan 1990). Recently a number of small asteroids have been discovered in Taurid-like—i.e., Encke-like—orbits (Asher *et al.* 1993), and these objects are considered to be of major concern from the point of view of the impact hazard on Earth (Bailey *et al.* 1994), as also hinted at by the similarity of radiants and orbits of the Tunguska object and of the Taurid meteoroids pointed out by Kresák (1978). In this paper we investigate the dynamics of these bodies in order to draw some conclusions as to their possible origin and final fate.

As a matter of fact, the origin of these bodies is somewhat puzzling. In particular, it is not easy to explain how P/Encke arrived at its present orbit since its aphelion distance has a low value ($Q = 4.1$ AU), which does not allow close encounters with Jupiter (Kresák 1979).

Carusi *et al.* (1981) studied numerically the orbit of P/Oterma, under the hypothesis that P/Encke had a similar orbit at some time in the past; the reason for this assumption is that both comets have a very high value (≥ 3) of the Tisserand parameter

$$T = \frac{a_J}{a} + 2\sqrt{\frac{a(1-e^2)}{a_J}}\cos i$$

(where a_J is the semimajor axis of the orbit of Jupiter,

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and a, e, and i are the semimajor axis, eccentricity, and inclination of the comet's orbit). Comets with such values of T tend to have dynamical behavior similar to that of P/Oterma, characterized by very effective close encounters with Jupiter in which temporary satellite captures can occur and by pre- and post-encounter orbits that are either totally exterior or totally interior with respect to that of the planet (Carusi and Valesecchi 1987). Some other notable examples of such comets, which because of their large perihelion distances ($q \geq 2$ AU) are being discovered more frequently now than in the past, are P/Gehrels 3, P/Gunn, P/Smirnova-Chernykh, P/Helin-Roman-Crockett, and P/Shoemaker-Levy 9.

By varying the initial conditions of the close encounter with Jupiter that brought P/Oterma into the orbit in which it was discovered, Carusi *et al.* tried to minimize both the perihelion and the aphelion distances of the post-encounter orbit, in order to bring them as close to those of P/Encke as possible. The smallest aphelion distance they could achieve was $Q = 4.49$ AU, and this means that other mechanisms, like nongravitational forces and encounters with the terrestrial planets (once the perihelion distance has been sufficiently reduced) are necessary to reach an Encke-like orbit. But these mechanisms would have virtually no time to operate effectively: Jupiter is able to strongly modify the orbit of a comet with $Q = 4.5$ AU and $e \leq 0.2$ in a very short time, of the order of a few revolutions, so that Carusi *et al.* concluded that it is unlikely for an Encke-like orbit to be obtained starting from a high-T short-period comet orbit like that of P/Oterma.

In this paper we find a different dynamical route connecting the present orbit of P/Encke to orbits typical of the Jupiter family of comets, which involves both secular resonances and close encounters with the terrestrial planets. However, the typical timescale turns out to be of several hundred thousand years, which is somewhat longer than what is usually assumed to be the physical age of the comet. We recall that Wetherill (1991) had examined the mechanisms capable of decoupling the aphelia of short-period comets from Jupiter, suggesting that encounters with the terrestrial planets and non-gravitational forces play a major role and coming to the conclusion that comets possibly contribute to the population of "Earth-approaching bodies of asteroidal appearance."

A second tempting hypothesis on the origin of P/Encke and of the Taurid objects is that of the fragmentation of a large parent body. Whipple and Hamid (1952) computed the secular motion of a number of photographic meteors and of P/Encke using Brouwer's theory (Brouwer 1947) and concluded that there should have been *two* parent bodies in similar orbits from which the observed stream particles should have been ejected at different epochs. One of the two parents was identified as P/Encke, and the other as "a body moving in an orbit of similar shape and line of apsides but somewhat greater aphelion distance, possibly a component of Comet Encke that split away at an unknown time in the past." The dimensions of the particles studied by Whipple and Hamid were estimated to be of the order of 1 cm, and the orbital analysis led to the conclusion that the ejections had taken place *not* close to perihelion, as is normal for ejections due to cometary activity, but *at larger distances,* close to 3 AU, probably because of impacts with asteroidal bodies.

Later studies of the orbital evolution of Taurid meteoroids have resorted to numerical integration to improve on the results by Whipple and Hamid; Jones (1986), using a restricted Sun–Jupiter–meteoroid model, found the age of the stream to be of the order of 0.1 Myr and found that it was not necessary to assume anymore that the separation of meteor particles from P/Encke had taken place away from perihelion. He also suggested that a more realistic modeling of the initial dispersion of the stream than the one that he had adopted might help to reduce the age estimate.

Steel *et al.* (1991), also in the framework of a restricted Sun–Jupiter–meteoroid problem, found results supporting the idea of Clube and Napier (1984) of a large progenitor comet that entered the inner solar system about 20,000 years ago and then, through a succession of splittings, gave origin to P/Encke, the zodiacal cloud, and the Taurid meteoroid complex.

Here we will not deal with issues related to the motion of dust particles or to the formation and age of the Taurid meteoroids; we will rather concentrate on the long-term dynamical evolution of P/Encke and of the small asteroids in Encke-like orbits.

We find that the secular dynamics, and in particular secular resonances, are sufficient to explain the existence of many bodies on Encke-like orbits, coming from the main asteroid belt and from the Jupiter family of comets; from a dynamical point of view the existence of a large parent body is therefore no longer necessary.

Conversely, in light of our result, we expect to find on Encke-like orbits both rocky and icy bodies (the former coming from the asteroid belt, and the latter from the Jupiter family of comets). In this region, then, the two populations mix up and can, in principle, exchange their entry and exit routes: a rocky body can end up on an orbit typical of the Jupiter family of comets, and an icy body can temporarily evolve to a low eccentricity orbit typical of the main asteroid belt.

Moreover, objects on Encke-like orbits have a surprisingly high probability of falling into the Sun, by having the eccentricity of their orbits pumped up to almost 1. Indeed, in our few-Myr integrations, 13 of the 21 bodies share this fate. We stress that these results could not have been found in the framework of the simple 3-body problem used by both Jones (1986) and Steel *et al.* (1991) since, to find them, it is indeed necessary to take into account a more realistic model of the Solar System in which the motions of the apse and nodal lines of planetary orbits are also present.

The remainder of this paper is structured as follows. In Section 2 we analyze in detail the dynamical evolution of P/Encke, pointing out the main features of the secular resonant dynamics in the Taurid region. In Section 3, starting from the numerical integrations of several real objects in Encke-like orbits, we discuss the origin, evolution, and possible end states of the Taurid population, addressing the questions related to its entry and exit routes. Section 4 discusses the possible genetic relationships among the objects, with particular emphasis on the one between P/Encke and 2212 Hephaistos. The conclusions then follow.

2. THE LONG-TERM DYNAMICS OF P/ENCKE

The motion of P/Encke has been integrated by Levison and Duncan (1994) over a time span of a few times 10^5 years. They found that the orbit of the comet would become sungrazing in both directions in time, within about -3×10^5 yr in the past and 5×10^4 years in the future. They also remarked that the orbital behavior of the comet is not understandable in the light of the now well-known mechanism by which comets on high-inclination, elongated orbits can become sungrazers, as pointed out by Bailey *et al.* (1992), during the high-eccentricity phase of the ω-cycle identified by Kozai (1962, 1979). In the case of P/Encke, indeed, the secular evolution of the orbital elements is quite smooth over a time span that is much larger than the typical Kozai time scale (i.e., the period of circulation of ω).

The smoothness of the secular behavior of P/Encke suggested to us that, in analogy with the cases of many high-eccentricity asteroids, the long-term evolution might be dominated by the effects of secular resonances with the apsidal motion of Jupiter and, especially, of Saturn. The information provided in the graphs presented by Levison and Duncan (1994) is largely insufficient to check this; we have therefore recomputed the motion of P/Encke over a comparable time span, using a Bulirsch–Stoer integrator and a purely gravitational model of the solar system containing all of the planets from Venus to Neptune. We have not modeled the nongravitational forces, as is customary in long-term integrations, since they are obviously not known outside the limited time span covered by observations. Their effect would presumably slightly add some more chaoticity to the motion, without changing the general features described below (i.e., the interplay between secular and/or mean motion resonances and the effects of close encounters with the terrestrial planets).

If our hypothesis were confirmed by the analysis of the results of the integration, this would be the first case found of a comet whose motion is determined by the presence of secular resonances close by.

Secular resonances play a fundamental role in the dynamics of asteroids. This was first conjectured by LeVerrier (1855), followed by Tisserand (1882) and Poincaré (1892); the first modern study was that by Williams (1969) and, following him, many research groups in Europe and Japan have explored in detail the various aspects of the problem (for a recent review see Froeschlé and Morbidelli 1994). We summarize in the following the basic features of secular resonant dynamics.

According to secular perturbation theories, the orbital elements of the planets oscillate with periods ranging from thousands to millions of years. If one restricts to the Sun–Jupiter–Saturn system, these changes are quasi-periodic with three basic frequencies: g_5 (the average precession rate of Jupiter's longitude of perihelion), g_6 (the average precession rate of Saturn's longitude of perihelion), and s_6 (the precession rate of the common line of nodes on the invariant plane of the system).

The planets also exert secular perturbations on any small body orbiting around the Sun and force the precession of its orbit; we denote by g the precession frequency of the small body's longitude of perihelion and by s the precession frequency of its node.

Moreover, the eccentricity and the inclination of the small body's orbit are forced to oscillate. The spectrum of the oscillations of e is characterized by three main terms, with frequencies $g - g_5$, $g - g_6$, and $2(g - s)$, while that of the inclination contains just the main terms, with frequencies $s - s_6$ and $2(g - s)$ (we recall that the term with frequency $2(g - s)$ is nothing but the already mentioned ω-cycle found by Kozai). The amplitude of these forced terms is roughly proportional to the inverse of the corresponding frequency; thus, the changes of e or i become particularly large approaching a resonance, i.e., when one of the frequencies in the spectrum becomes close to 0.

We call the resonance where $g = s$, the Kozai resonance and the resonances involving the frequencies of perihelia or nodes of the small body and of the planets secular resonances. In particular, following Williams' notation, the resonances with, respectively, $g = g_5$, $g = g_6$, and $s = s_6$ are usually called ν_5, ν_6, and ν_{16}.

Armed with this knowledge, understanding the secular behavior of a numerically integrated orbit then becomes an easy matter, and the discussion below on the secular evolution of P/Encke's orbit is just an example of that. One simply plots the evolution with respect to time of the critical angles of each resonance and correlates them to the changes in e and i. The critical angles are ω for the Kozai resonance, $\tilde{\omega} - \tilde{\omega}_J$ for the ν_5 resonance, $\tilde{\omega} - \tilde{\omega}_S$ for the ν_6 resonance, and $\Omega - \Omega_S$ for the ν_{16} resonance; here $\tilde{\omega}_J$ (respectively $\tilde{\omega}_S$) is the longitude of perihelion of Jupiter (respectively Saturn) and Ω_S is the longitude of node of

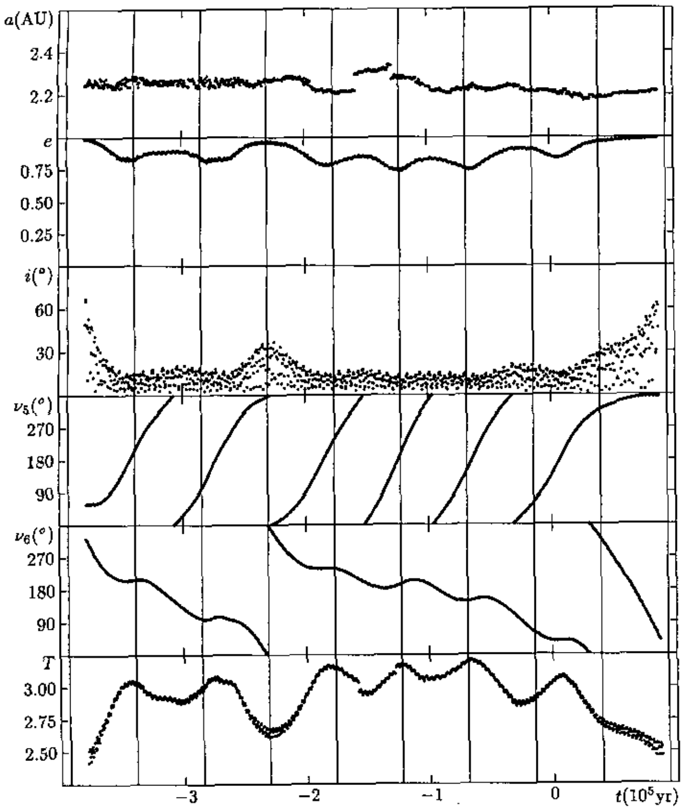

FIG. 1. The time evolution of (top to bottom) the semimajor axis a, the eccentricity e, the inclination i, the critical angles of the ν_5 and ν_6 secular resonances, and the Tisserand parameter T, for the orbit of P/Encke; vertical lines mark the epochs when the perihelia of Jupiter and Saturn are aligned.

Saturn. The nodes should always be computed with respect to the invariant plane.

Figure 1 shows the secular behavior of the orbit of P/Encke. We first remark that the semimajor axis a is practically constant during the whole time span integrated. Indeed, the comet never encounters Jupiter (the aphelion distance is too small), and encounters with the Earth and Mars produce just small chaotic variations of a, yielding a kind of random walk. The eccentricity e shows "smooth" oscillations, as already remarked by Levison and Duncan, increasing to 1 at $t \sim -3 \times 10^5$ years and at $t \sim 5 \times 10^4$ years, finally throwing the comet into the Sun.

What is the cause of these "smooth" oscillations? Let us consider the critical angles ω, $\tilde{\omega} - \tilde{\omega}_J$, and $\tilde{\omega} - \tilde{\omega}_S$, affecting the behavior of the eccentricity. The perihelion argument ω (not shown in Fig. 1) circulates quite rapidly and causes just a small and short periodic oscillation of e; at the scale of the figure, this small oscillation is comparable to the thickness of the curve. The critical angle of the ν_5 resonance $\tilde{\omega} - \tilde{\omega}_J$ circulates counterclockwise (i.e., $g > g_5$) with a period of about 5×10^4 years; these circulations are perfectly correlated with the oscillations of the eccentricity, each passage $\tilde{\omega} - \tilde{\omega}_J = 0$ corresponding to a maximum of e. The critical angle of the ν_6 resonance $\tilde{\omega} - \tilde{\omega}_S$ circulates clockwise (i.e., $g < g_6$) with a much longer period (2.5×10^5 years for the slowest circulation); this circulation is perfectly correlated with the superimposed large oscillation of the eccentricity, with the supermaxima corresponding to $\tilde{\omega} - \tilde{\omega}_S = 0$. When the oscillations of e due to $\tilde{\omega} - \tilde{\omega}_J$ and to $\tilde{\omega} - \tilde{\omega}_S$ are in phase the eccentricity can reach 1 and then the comet hits the Sun. We remark also that, just before the collision with the Sun, the critical angle of the ν_5 resonance changes direction of motion, showing that the orbit enters into the resonance.

To ease the interpretation of the eccentricity modulation, in Fig. 1 we have marked with vertical lines the epochs when the perihelia of Jupiter and Saturn are aligned, which is, as we have just seen, a necessary condition for reaching the highest values of the eccentricity.

Finally, for what concerns the inclination i, we note that it changes rapidly with the frequency of circulation of 2ω. The minimum values of i are approximately the same at each oscillation, while the maximum values are modulated following the evolution of the eccentricity. This is due to the fact that ν_5 and ν_6 change the value of $\sqrt{a(1-e^2)}\cos i$ (the quasi-constant of motion during Kozai's cycle) and the amplitude of variation of i along a Kozai cycle depends precisely on the value of this quantity. Conversely, the motion of the critical angle $\Omega - \Omega_S$ does not particularly influence the behavior of i in the case of P/Encke and therefore is not shown in Fig. 1.

In the light of these results, the puzzling problem of the origin of P/Encke cannot be solved in a definitive way, although we now understand the reason for its sun-hitting fate. The next section is devoted to the analysis of the dynamical behavior of asteroids on Encke-like orbits. From an analysis of the behavior of these asteroids some conjectures on the origin of P/Encke can be drawn; we discuss the several possibilities in Section 4.

3. THE DYNAMICS OF ASTEROIDS ON ENCKE-LIKE ORBITS

Asher *et al.* (1993) have recently called attention to the presence of a number of small asteroids in the Taurid complex, suggesting that this should have important implications for the history of the Taurid complex itself. They used a reduced version of the well-known D-criterion (Southworth and Hawkins 1963), involving in this case only a, e, and i, i.e.,

$$D^2 = \left(\frac{a_1 - a_2}{3}\right)^2 + (e_1 - e_2)^2 + \left(2\sin\frac{i_1 - i_2}{2}\right)^2,$$

to find asteroids with orbits similar to those of Taurid

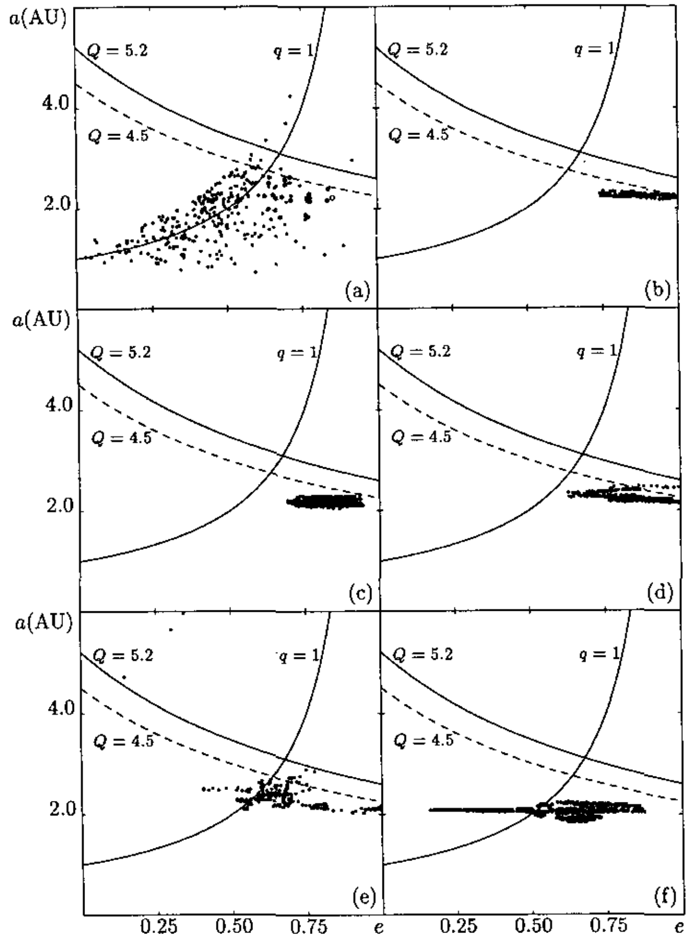

FIG. 2. Planet crossing asteroids in the a–e plane. The curves in each box represent orbits fulfilling the condition given by the corresponding label; above the dashed curve labeled $Q = 4.5$ nearly tangent close encounters with Jupiter are possible, and above the curve labeled $Q = 5.2$ these encounters start to be crossing. Below the curve labeled $q = 1$ the orbits cross that of the Earth. In (a) are shown the Apollo–Amor–Aten asteroids discovered until 20 February 1994; the large open circle represents P/Encke and the large dots denote "Taurid complex" asteroids (see text). In (b), (c), (d), (e), and (f) are shown the orbital evolutions of, respectively, P/Encke, 1990 SM, 2212 Hephaistos, 4486 Mithra, and 4341 Poseidon; the individual dots are spaced by 2000 yr.

meteoroids, and found that two groups are present, one in which the apse line is more or less aligned with that of P/Encke, and one in which the approximate alignment is with the apse of 2212 Hephaistos.

Figure 2a shows an a–e diagram containing all of the Apollo–Amor–Aten asteroids discovered until 20 February 1994, taken from a list kindly provided to us by D. Steel. The large open circle represents the present position of P/Encke; the large dots denote the asteroids contained in the updated list of "Taurid complex" members taken from Steel (1994), who selected the bodies with $D < 0.25$ with respect to P/Encke's orbit. Note that these bodies tend to be on the right of the diagram, in the region of the highest eccentricities. We call the region with $D < 0.25$ from P/Encke the "Taurid region."

We have integrated almost all of the objects in Steel's list, except 5025 P-L and 1993 KA_2, with the same integrator and dynamical model used for P/Encke; the integration time span extended over about 1 to 2 Myr in the past and in the future. Table I lists the objects integrated and summarizes their dynamical evolution and final fates.

The accuracy with which the orbits of the asteroids that we have integrated are known varies widely, since in the list there are both numbered and unnumbered asteroids, and some of the latter have been observed only over very short arcs. However, we just used them as test orbits to explore the typical dynamics over Myr time scales in the Taurid region; from this point of view, the results would have not qualitatively changed if we had used some fictitious objects spread out in the same box of orbital element space.

Starting from the considerations of Asher *et al.* (1993), one could expect to find rather similar dynamical evolution, at least on a short timescale, for most of these bodies. Actually, we have been surprised by the wide variety of behavior. We are not going to delve here into the details of the outcome of each integration; to do so would be inappropriate, since the orbits are all strongly chaotic. Instead, we extract from our numerical data some general considerations concerning the dynamical evolution of objects in the Taurid region and the entry and exit routes into/from it.

The general picture that we get is that the population which currently occupies the Taurid region is transient, with entry and exit routes that connect it with both the asteroid main belt and the Jupiter family of comets.

Let us first examine the behavior of 1990 SM, an object that can be considered a "typical Taurid," since it is deeply buried between the ν_5 and the ν_6 resonance, with the aphelion well decoupled from the orbit of Jupiter; its orbital evolution is shown in Fig. 3.

As it is possible to see, the frequency of the longitude of perihelion of this object is more or less halfway between g_5 and g_6, and the encounters with the terrestrial planets are not effective enough to displace it toward either of the associated resonances, so that the eccentricity continues to oscillate rather regularly, with maxima well correlated with some of the alignments of the perihelion longitudes of Jupiter and Saturn. Another way to look at the behavior of this object is to consider its evolution in the a–e diagram (see Fig. 2c), where it wanders within the Taurid region, never increasing its eccentricity to a value sufficient to undergo nearly tangent encounters with Jupiter or to become a sungrazer; for comparison, consider the same plot for P/Encke, given in Fig. 2b.

To illustrate the entry and exit routes we show and

TABLE I
The Taurid Objects Integrated for This Paper, Listed in Order of Increasing Current Semimajor Axis

Name		a	e	i	Dynamics	Fate
5143	Heracles	1.83	0.771	9.2		
4341	Poseidon	1.84	0.679	11.9	ν_6: $2 < t < 12$	
2101	Adonis	1.88	0.764	1.4		
	1991 GO	1.96	0.662	9.7	4/1 & ν_5: $t > 4$	$t_{sun} = 6.0$
4183	Cuno	1.98	0.637	6.8		$t_{sun} = 1.8$
	1990 SM	2.16	0.775	11.6		
	1991 AQ	2.16	0.769	3.2	Near ν_6 for $t < -4$	$t_{sun} = -7.7$
2212	Hephaistos	2.17	0.833	11.8	ν_6: $-10 < t < -6$; ν_5: $t > 0$	$t_{sun} = -10.2$ $t_{sun} = 1.1$
2201	Oljato	2.18	0.711	2.5	ν_6: $-1 < t < 1$	$t_{sun} = 3.6$
4486	Mithra	2.20	0.662	3.0	ν_6: $-1 < t < 1$	$t_{hyp} = -5.5$ $t_{sun} = 1.8$
	P/Encke	2.22	0.850	11.9	Near ν_6	$t_{sun} = -3.8$ $t_{sun} = 0.9$
6063	1984 KB	2.22	0.764	4.8	ν_6: $-2 < t < 0$	$t_{sun} = -2.7$
	1991 BA	2.24	0.682	2.0	ν_6: $t < 0$	$t_{sun} = -3.8$ $t_{sun} = 1.2$
	1991 EE	2.25	0.624	9.8	ν_6: $t > 1$	
5731	1988 VP_4	2.26	0.653	11.7	3/1 & ν_6: $t > 5$	$t_{sun} = 14.7$
4197	1982 TA	2.30	0.773	12.2	ν_6: $0 < t < 8$	
	1991 TB_2	2.40	0.836	8.7	ν_6	$t_{sun} = -2.5$ $t_{sun} = 0.3$
	1990 TG_1	2.48	0.692	9.1	3/1: $-1 < t < 0$	$t_{hyp} = -6.5$ $t_{hyp} = 4.2$
4179	Toutatis	2.51	0.640	0.5	ν_6: $t < -7$	$t_{hyp} = 6.4$
	1990 HA	2.58	0.693	3.9	ν_6: $-4 < t < -3$; 3/1 & ν_5: $-3 < t < 1$	$t_{sun} = -4.1$ $t_{hyp} = 4.5$
	6344 P-L	2.62	0.641	4.6	9/4: $-10 < t < -9$; 5/2: $-2 < t < -1$; 5/2: $0 < t < 1$; ν_5: $t > 2$	$t_{hyp} = 2.4$
	1983 LC	2.63	0.710	1.5	ν_6: $t > 6.5$	$t_{hyp} = -1.1$ $t_{sun} = 8.1$

Note. The first column gives the name and designation. Columns 2 to 4 give the present values of a, e, and i. Column 5 describes the major dynamical features happening when the body does not have the typical Taurid behavior, in which a undergoes a gentle random walk due to encounters with Earth and Venus, and e oscillates under the effect of the ν_5 and ν_6 terms. Column 6 gives the final fate, when it is either the fall into the Sun at time t_{sun}, or the hyperbolic ejection, at time t_{hyp}, caused by an encounter with one of the outer planets (most often Jupiter), after a more or less prolonged stay in the Jupiter family and, in some cases, also in orbits typical of the long-period comets. Units are Astronomical Units for column 2, degrees for column 4, and 10^5 years for columns 5 and 6.

discuss three nice examples of evolution along them, provided by our numerical integrations of the asteroids 2212 Hephaistos, 4486 Mithra, and 4341 Poseidon. Of course, these orbits being chaotic, these are not the real evolutions of Hephaistos, Mithra, and Poseidon, but must just be considered examples of possible evolutions, not only for these bodies but also for any body currently in the Taurid region.

Figure 4 shows the evolution of 2212 Hephaistos. We are interested here in the backward integration. The random walk of the semimajor axis, due to the subsequent small changes caused by close encounters with the inner planets (mostly the Earth), takes Hephaistos closer to the ν_6 resonance (located at $a \sim 2.4$ AU at Hephaistos' inclination) and finally into it. Indeed, looking at the evolution of the critical argument of the ν_6 resonance, i.e., $\tilde{\omega} - \tilde{\omega}_S$, one can see that it slows down its circulation and finally enters into libration. Consequently the eccentricity finally shows a long-term oscillation, in which it first diminishes to 0.65 (in correspondence with $\tilde{\omega} - \tilde{\omega}_S = 180°$ at $t \sim -8.5 \times 10^5$ years) and then grows to 1. Since the semimajor axis is about 2.4 AU, the aphelion distance at the end of

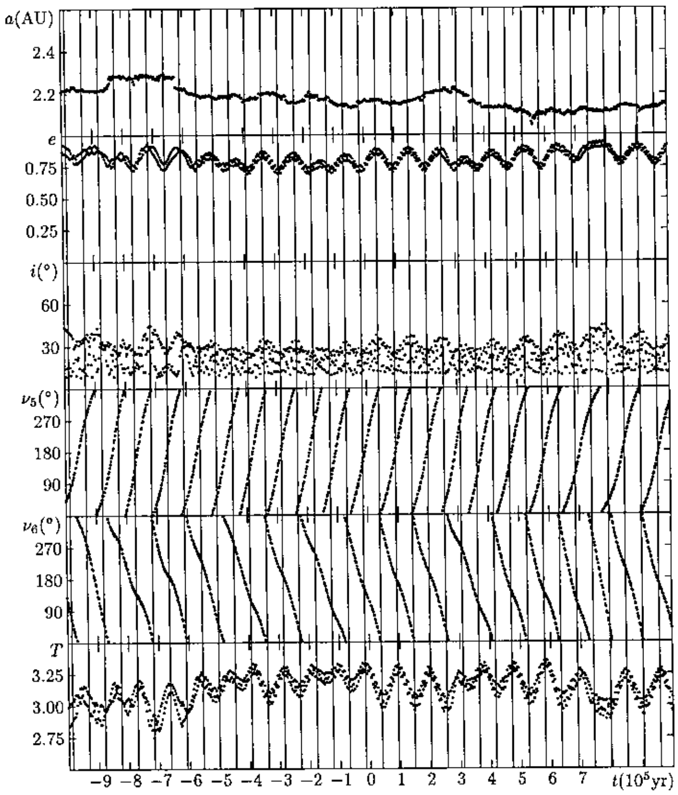

FIG. 3. The time evolution, in the same arrangement of Fig. 1, of the orbital elements of 1990 SM.

the backward integration is so large that encounters with Jupiter are possible. These encounters actually do not take place, since the eccentricity is already so high that its further oscillations are sufficient to drive the object into the Sun before Jupiter really has a chance to decouple it from the ν_6 resonance.

However, this is not the only case of transfer from the Taurid region into the Jupiter family present in our integrations and is not the most typical either. An inspection of Table I shows a significant number of hyperbolic ejections, all of which take place after a certain time spent in the Jupiter family. These cases tend to occur, not surprisingly, in the bottom part of the table, where the starting semimajor axes—and therefore, for the same eccentricity, the aphelion distances—are larger; among them a nice case to discuss is that of 4486 Mithra (see Fig. 5), whose starting semimajor axis is very close to that of P/Encke.

As it is possible to see in the figure, while the future orbital evolution of 4486 Mithra is similar to those of P/Encke and 2212 Hephaistos, its past evolution is characterized by a rotation of the perihelion line slower than that of Saturn, accompanied by a systematic growth (going backward in time!) of the semimajor axis. As a result, the eccentricity remains at levels comparable to the current one, but the aphelion distance grows until encounters with Jupiter become possible; when this happens, around $t \sim -3.5 \times 10^5$ years, its Tisserand parameter is between 2.8 and 3, and its eccentricity is about 0.7, values typical of Jupiter family members. The subsequent evolution, up to the hyperbolic ejection, is typically cometary.

It is important to note that this dynamical channel from the Taurid region to the Jupiter family, in the two variants corresponding to the cases of 2212 Hephaistos and 4486 Mithra, has essentially nothing to do with the one that Carusi *et al.* (1981) explored. In that case the orbits studied were of comets just transferred from outside to inside the orbit of Jupiter as a consequence of a very slow encounter with the planet; the Tisserand parameter was therefore high, and the eccentricity rather low (for a semi-quantitative argument giving the likely values of the eccentricity in this case, see Valsecchi 1992). These orbital characteristics make encounters with Jupiter extremely frequent. In the present case the "gate" to/from the Jupiter family is at high eccentricity ($e \geq 0.7$, see also Figs. 2d and 2e), and the frequency of encounters with the planet on orbits of such high eccentricities is much lower than in the low-e case. Consequently, comets close to this "gate" are likely to have evolved there not directly from the region between Jupiter and Saturn after a nearly tangent encounter, but

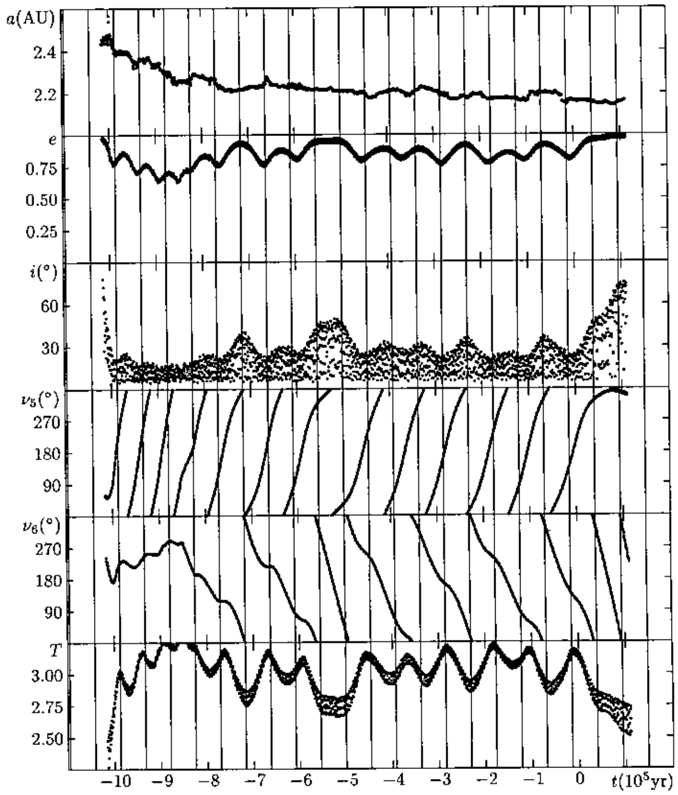

FIG. 4. The time evolution, in the same arrangement of Fig. 1, of the orbital elements of 2212 Hephaistos.

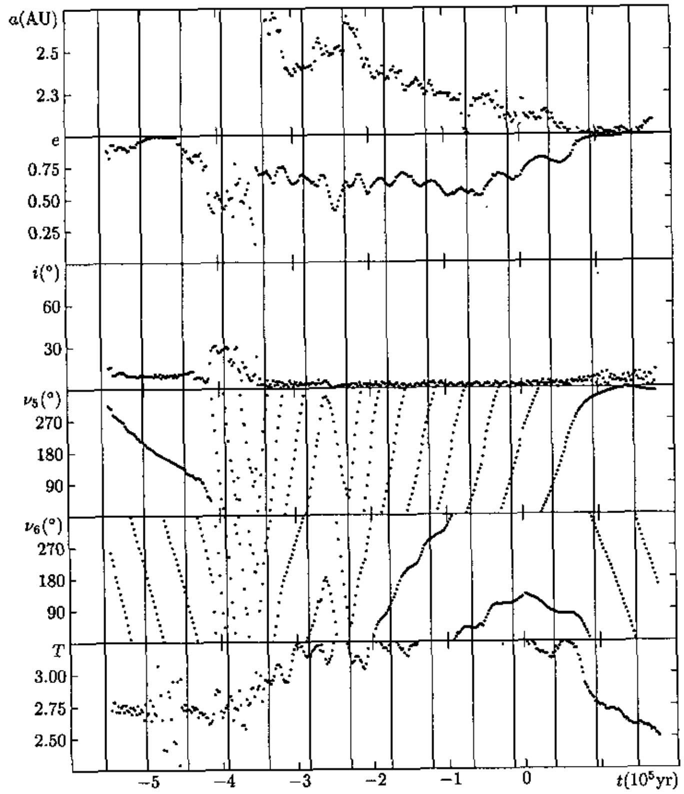

FIG. 5. The time evolution, in the same arrangement of Fig. 1, of the orbital elements of 4486 Mithra.

at a rather slower pace, through a route involving higher eccentricities. Moreover, due to the secular perturbations, the Tisserand parameter is not even approximately conserved, and we can therefore conclude that the current value of T for P/Encke is not necessarily close to the one that the comet had when it was in the Jupiter family.

Let us now come to the asteroidal route. Figure 6 shows the evolution of 4341 Poseidon. We are interested here in the time span from 9×10^5 to 1.6×10^6 years. At 9×10^5 years Poseidon is close to the inner edge of the main belt, with semimajor axis about 2.1 AU and eccentricity 0.2 (inclination about 10°). However, Poseidon being in the ν_6 resonance, in a few hundred thousand years the libration of the critical angle $\tilde{\omega} - \tilde{\omega}_S$ about 0 takes the eccentricity to 0.8, i.e., to an orbit typical of the Taurid region. Then, close encounters with the inner planets change the semimajor axis, decoupling the orbit from the resonance.

All of the above evolutions show that the ν_6 secular resonance behaves like a "shuttle" in eccentricity, as long as the semimajor axis remains constant; the random walk of the semimajor axis, induced by close encounters with the planets, can cause the orbit to enter or exit the resonance at specific times, thereby making evolution between orbits of widely different eccentricities possible. We note that, in principle, other resonances, such as the 3/1 or 5/2 mean motion resonances, could also play this shuttle role for the eccentricity (Froeschlé *et al.* 1995); however, we did not find any such case in our integrations, suggesting that this is a less probable mechanism.

In Figs. 2b, 2c, 2d, 2e, and 2f are represented, in the a–e plane, all of the orbital evolutions that we have discussed so far, over the relevant time spans. One can see that objects coming from the two routes outlined above, i.e., coming either from the Jupiter family of comets or from the asteroïd main belt through the ν_6 shuttle, can wander all over the Taurid region, as comparison with Fig. 2a shows. Outside the resonance regions this wandering on the a–e plane is comparatively slower, and this is why we observe the majority of the objects in the Taurid region in this nonresonant phase, rather than in the resonant transport one.

We come now to the exit routes from the Taurid region. The dynamical entry route exemplified by 2212 Hephaistos and 4486 Mithra can be reversed: an object in the Taurid region can evolve to a Jupiter tangent/crossing orbit and be removed from the vicinity of the Taurid region by close encounters with the giant planet. A second exit route is given by solar collisions: the secular dynamics can lead the eccentricity to 1, with a small enough semimajor axis so

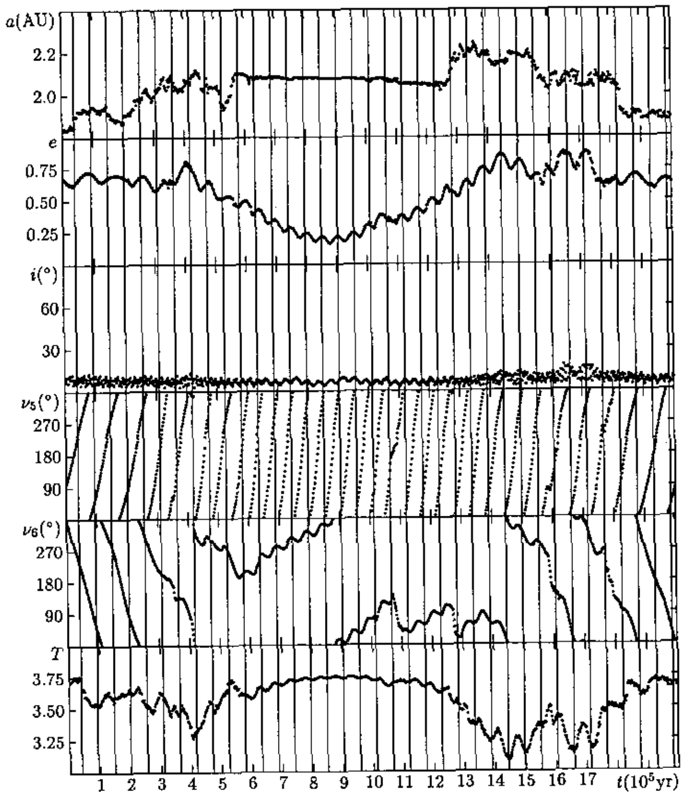

FIG. 6. The time evolution, in the same arrangement of Fig. 1, of the orbital elements of 4341 Poseidon.

that encounters with Jupiter are avoided, and throw the object into the Sun. This seems to be a fairly common fate according to our numerical integrations (see Farinella *et al.* 1994 for statistics).

On the other hand, the entry route exemplified by 4341 Poseidon cannot be reversed to be considered as an exit route. Indeed, an object in a resonance can decrease the eccentricity to a typical main belt value but, since close encounters with the planets are then no longer possible, cannot leave the resonance. By consequence, its eccentricity will eventually increase back to the starting value; this can be seen also in our full integration of Poseidon (Fig. 6).

Coming back to the mixing of the dynamical paths of Hephaistos, Mithra, and Poseidon, it implies that, at least in principle, we can expect to find in the Taurid region both rocky and icy bodies; this fits nicely with the observed presence of two components in the population of Taurid fireballs, an "asteroidal," higher density one, and a "cometary," lower density one (Ceplecha 1994, personal communication). Moreover, we could also find rocky bodies on orbits typical of the Jupiter family of comets and icy bodies (possibly in dormant phases) temporarily in orbits resembling those of main belt asteroids. In the latter case, however, these icy bodies can only be in resonant orbits since, as explained before, there is no efficient mechanism for quitting a resonance during the low eccentricity phase.

We stress that, because the dynamical channel linking the main belt to the Jupiter family passes through the Taurid region, it is unnecessary to assume, as done by Wetherill (1991) and by Festou *et al.* (1993), that the bodies of asteroidal appearance observed in short-period comet orbits are defunct comets: these bodies may simply be former main belt asteroids.

These possibilities point strongly to the need for physical observations of both near Earth asteroids and main belt asteroids which are known to be close to resonances.

4. IS THERE A GENETIC CONNECTION AMONG P/ENCKE AND THE TAURID ASTEROIDS?

We come now to discuss the long-standing problem of the origin and evolution of P/Encke. The results described in the previous section show that it is possible for a comet to detach its aphelion from Jupiter due only to gravitational causes (i.e., secular resonances); unfortunately, this dynamical path takes a much longer time than one would like. Actually, P/Encke must not be a young comet; Kamel (1991) has critically reexamined the available observational data and has concluded that there is no evidence for a secular fading, contrary to various claims previously appearing in the literature. Moreover, recent modeling of the physical evolution of P/Encke, based on the numerical integrations of Levison and Duncan (1994), suggests that the total active lifetime of the comet may have been as

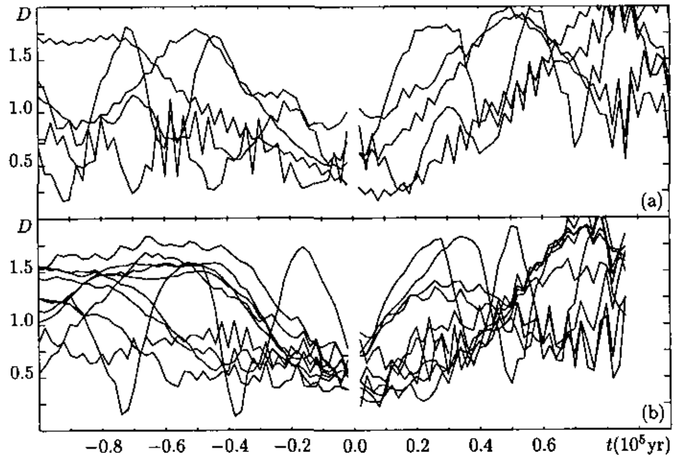

FIG. 7. The time evolution of the D parameter for pairs made of P/Encke and members of its "group" (a) and of 2212 Hephaistos and members of its "group" (b).

long as 65,000 revolutions, i.e., a few times 10^5 years (Whipple 1994), in reasonable agreement with the timescale of our transfer of 4486 Mithra from the Jupiter family into the Taurid region.

Another possibility already discussed in the literature (Clube and Napier 1984) is that a giant comet of the Jupiter family could have ended in an Encke-like orbit and have suffered afterwards a hierarchical fragmentation. According to this scenario, the injection of the progenitor in the Taurid region should have occurred about 20,000 years ago, and the detachment of P/Encke from the parent body about 3000 years ago. There are at least two problems with this hypothesis: first, the 30-km devolatilized remnant that it predicts has not yet been discovered; second, and more important from our point of view, we have found no evidence of any injection mechanism from the Jupiter family into the Taurid region that can work on a timescale of only 20,000 years.

Nevertheless, the possibility of a genetic relationship between P/Encke and some of the asteroids in the Taurid region is intriguing, and we have examined the output of our integrations, keeping this in mind. The already-mentioned finding by Asher *et al.* (1993) that the longitudes of perihelia of asteroids in the Taurid region tend to cluster in two groups, one aligned with the orbit of P/Encke, and the other with that of 2212 Hephaistos, can be viewed as an argument in favor of the common large progenitor hypothesis.

We have therefore checked the similarity of the orbits in the past making use of the D-criterion (the full version of it, involving also $\tilde{\omega}$ and Ω, see Southworth and Hawkins 1963). In particular, we have computed the backward time evolution of D for pairs constituted by P/Encke and members of its "group" (see Fig. 7a) and for pairs constituted

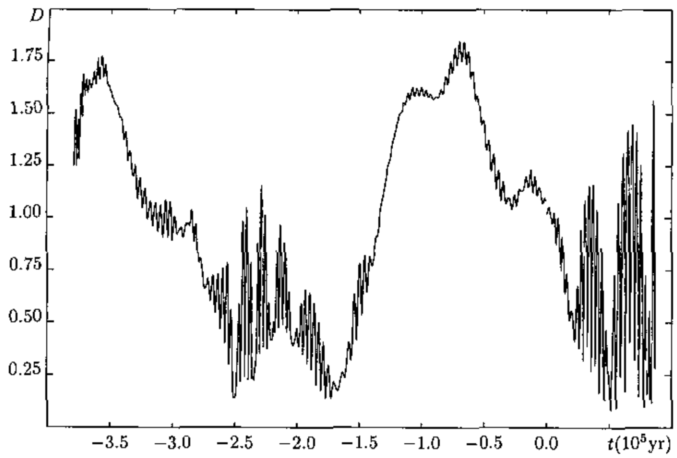

FIG. 8. The past evolution of the D parameter for the pair made of P/Encke and 2212 Hephaistos.

by 2212 Hephaistos and members of its "group" (Fig. 7b). Current values of D are 0.4 to 0.5, corresponding to relative velocities at a hypothetical recent ejection of many km/sec (Jopek 1993), too high for any reasonable model of hierarchical fragmentation. If the genetic relationship were true, we would expect a decrease of D in the past, at least in the first few 10^4 years. On the contrary, we always find a rapid growth of D, going up even to 1 in a typical timescale of 50,000 years. This is because the precession rate of $\tilde{\omega}$ for each asteroid depends on the values of its e, i, and especially a; the grouping in a–e–i space found by Asher *et al.* (1993) is simply not compact enough to warrant a similar overall precession rate, and therefore the various apse lines start to disperse quickly.

We have also examined the behavior of D for P/Encke and the members of the Hephaistos group. Figure 8 shows the result for the pair P/Encke–2212 Hephaistos; we discuss this case because Hephaistos is probably the largest among the Taurid region asteroids and, adopting a variant of the hypothesis put forward by Asher *et al.* (1993), may be considered the devolatilized remnant of a larger parent. As it is easy to see, the ~80° difference in the longitudes of perihelia shows up clearly in the high starting value of D (which, incidentally, is not larger than the current typical values for P/Encke and members of its own group, see Fig. 7a). Continuing backward D becomes even larger, since the difference of the perihelion longitudes increases, and starts to diminish only beyond about 100,000 years, when the different precession rates of the apse lines work in the right way. D reaches minimum values of ~0.14 at around −170,000 and −250,000 years when the apse lines start to separate again.

This behavior of D can be understood looking at the parts of Figs. 1 and 4 showing the evolution of the apse lines relative to that of Saturn. Clearly the relative precession rate of the apse line of P/Encke is slower, because of its greater proximity to the ν_6 resonance, due in turn to the slight difference of the values of a of the two bodies (a_H = 2.17 AU for 2212 Hephaistos and a_E = 2.22 AU for P/Encke). If, say, the value of a_H in the past were raised to become nearly equal to a_E, then 2212 Hephaistos would go closer to the ν_6 resonance, and the motion of its apse line relative to that of Saturn would also slow down, so that the separation of orbits that we observe reflected in the evolution of D (see Fig. 8) would possibly not take place. Actually, this speculative scenario does not work, and we cannot hope to receive much help from encounters with either the Earth or Venus, since the timescales for encounters with either of the two planets deep enough to bring the semimajor axis of one of the two objects to coincide with that of the other are in excess of several Myr. We are therefore left with the conclusion that even a scenario in which P/Encke was detached in the recent past from a parent body, whose larger remnant is now 2212 Hephaistos, is faced with the timescale problem.

5. DISCUSSION AND CONCLUSIONS

The orbital integrations discussed in this paper give us a global view of the dynamics of objects in Taurid-like orbits. These objects occupy the region of the phase space approximately bounded by the 4/1 and 3/1 mean motion resonances and by the ν_5 and ν_6 secular resonances.

The 3/1 and the ν_6 resonances have been shown to be capable of extracting asteroids from the main belt and to increase their eccentricities up to values that allow encounters with the terrestrial planets (see e.g., Wisdom 1983, 1985, Farinella *et al.* 1993). The associated eccentricity cycles are slow enough to allow these planets, and especially the Earth, to perturb significantly the orbit of the resonant asteroids, displacing them from the resonance, so that they end up in the Taurid region, where the evolution of the elements is slower and from which the bodies exit either because of further encounters with the terrestrial planets or because they fall onto the Sun.

Our integrations show that the ν_6 resonance can connect the Taurid region also to the Jupiter family of comets, as exemplified by the backward integrations of 2212 Hephaistos and 4486 Mithra; it is likely that P/Encke arrived at its present orbit through this channel, although the numerical reconstruction of the actual past motion is impossible due to its chaoticity, as well as to our ignorance of the direction and strength of the nongravitational forces outside the observed time span.

This overall picture suggests a word of caution in evaluating theories of the origin of the bodies in the Taurid complex that imply genetic relationships like the hierarchical fragmentation of a large parent comet; our results do not speak *directly* against such a view, but indicate that there are efficient dynamical mechanisms that cause the injection of bodies into the Taurid region from both the main asteroid belt and the Jupiter family of comets. In the absence of a mechanism to bring many small bodies into a particular region of the orbital elements' space, it is tempting to reduce the problem to that of bringing there just one large body and then letting it fragment to give rise to the small ones, but if the transport mechanism exists, then such an approach seems less justified, at least on dynamical grounds.

Coming to the groupings of apsidal lines around those of P/Encke and 2212 Hephaistos pointed out by Asher *et al.* (1993), we find no reason why they should be of dynamical origin, since in our integrations the groupings disappear very quickly. This may then mean that the groupings are due either to chance or to some selection effects worth investigating; a third possibility is that there *has been* the hierarchical fragmentation of one or two larger bodies that has given origin to the Taurid complex asteroids, but then this splitting must have taken place *very* recently, since the time for spreading the apse lines in our integrations is of the order of a few thousand years; however, this would imply ejection speeds of the order of many km/sec, which is very hard to believe.

In summary, we find that numerical integrations over Myr time scales, using a model of the solar system with all of the planets from Venus to Neptune, show that:

- the long-term dynamics of P/Encke and of the asteroids in the Taurid complex is determined essentially by the interplay between close encounters with the inner planets and the influence of nearby resonances, both mean motion ones, i.e., the 4/1 and the 3/1, and secular ones, i.e., the ν_5 and, especially, the ν_6;
- the Taurid region of a–e–i space, i.e., that occupied by objects in Encke-like orbits, is bounded by the abovementioned resonances, which connect it to the possible sources of the objects found there;
- the 3/1 mean motion resonance and especially the ν_6 secular resonance connect rather efficiently, along paths at almost constant a and widely varying e, the low-eccentricity orbits typical of main-belt asteroids to the Taurid region;
- the ν_6 is also capable, again along a path at nearly constant a and variable e, to connect the Taurid region with cometary orbits of the Jupiter family that are either tangent in aphelion or crossing the orbit of the planet;
- the two previous points show that there is a dynamical path connecting the main asteroid belt to the Jupiter family of comets, passing through the Taurid region;
- small variations of a, induced by close encounters with the inner planets, are responsible for the displacement from resonance into the Taurid region, of further slow wandering within the region itself, and of possible further reinjection into resonance, in which case the object either is transferred to an orbit typical of the Jupiter family of comets, and then presumably ejected by encounters with the giant planet, or makes a round trip in orbital element space toward the main belt, where its residence is only temporary because there are no efficient mechanisms there to detach its orbit from the resonance;
- while residing in the Taurid region, or while traveling in orbital elements' space to/from it, small bodies have a surprisingly high probability of ending up in the Sun, a fate that is more probable than the hyperbolic ejection due to planetary encounters and, to an even larger extent, than a collision with the inner planets.

The population of the Taurid region thus appears to reside there during a transient stage of its evolution toward fall onto the Sun or ejection from the inner solar system. It is likely that P/Encke arrived there along the ν_6 path discussed before, although the question of its ability to survive for the long time span implied is still open. Physical observation of Taurid complex asteroids are necessary in order to be able to understand both the individual histories and that of the whole complex.

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REFERENCES

Asher, D. J., S. V. M. Clube, and D. I. Steel 1993. Asteroids in the Taurid Complex. *Mon. Not. R. Astron. Soc.* **264**, 93–105.

Bailey, M. E., J. E. Chambers, and G. Hahn 1992. Origin of sungrazers: A frequent cometary end-state. *Astron. Astrophys.* **257**, 315–322.

Bailey, M. E., S. V. M. Clube, G. Hahn, W. M. Napier, and G. B. Valsecchi 1994. Hazards due to giant comets: Climate and short-term catastrophism. In *Hazards Due to Comets and Asteroids* (T. Gehrels, Ed.), pp. 479–533. Univ. of Arizona Press, Tucson.

Brouwer, D. 1947. Secular variations of the elements of Encke's comet. *Astron. J.* **52**, 190–198.

Carusi, A., and G. B. Valsecchi 1987. Dynamical evolution of short-period comets. In *Interplanetary Matter* (Z. Ceplecha and P. Pecina, Eds.), pp. 21–28. Czechosl. Acad. Sci. Publications, Prague.

Carusi, A., Ľ. Kresák, and G. B. Valsecchi 1981. Perturbations by Jupiter of a chain of objects moving in the orbit of Comet Oterma. *Astron. Astrophys.* **99**, 262–269.

Clube, S. V. M., and W. M. Napier 1984. The microstructure of terrestrial catastrophism. *Mon. Not. R. Astron. Soc.* **211**, 966–968.

Farinella, P., Ch. Froeschlé, and R. Gonczi 1993. Meteorites from the asteroid 6 Hebe. *Cel. Mech. Dynam. Astron.* **56**, 287–305.

Farinella, P., Ch. Froeschlé, Cl. Froeschlé, R. Gonczi, G. Hahn, A. Morbidelli, and G. B. Valsecchi 1994. Asteroids falling onto the Sun. *Nature* **371**, 314–317.

FESTOU, M. C., H. RICKMAN, AND R. M. WEST 1993. Comets. II. Models, evolution, origin and outlook. *Astron. Astrophys. Rev.* **5,** 37–163.

FROESCHLÉ, CH., AND A. MORBIDELLI 1994. The secular resonances in the Solar System. In *Asteroids, Comets, Meteors 1993* (A. Milani, M. Di Martino, and A. Cellino, Eds.), pp. 189–204. Kluwer Academic, Dordrecht.

FROESCHLÉ, CH., G. HAHN, R. GONCZI, A. MORBIDELLI, AND P. FARINELLA 1995. Secular resonances and the dynamics of Mars-crossing and near-Earth asteroids. *Icarus* **117,** 45–61.

JONES, J. 1986. The effect of gravitational perturbations on the evolution of the Taurid meteor stream complex. *Mon. Not. R. Astron. Soc.* **221,** 257–267.

JOPEK, T. J. 1993. Remarks on the meteor orbital similarity *D*-criterion. *Icarus* **106,** 603–607.

KAMEL, L. 1991. The evolution of P/Encke's light curve—No secular fading, a vanishing perihelion asymmetry. *Icarus* **93,** 226–245.

KOZAI, Y. 1962. Secular perturbations of asteroids with high inclination and eccentricities. *Astron. J.* **67,** 591–598.

KOZAI, Y. 1979. Secular perturbations of asteroids and comets. In *Dynamics of the Solar System* (R. L. Duncombe, Ed.), pp. 231–237. Reidel, Dordrecht.

KRESÁK, Ľ. 1978. The Tunguska object: A fragment of comet Encke? *Bull. Astron. Inst. Czechosl.* **29,** 129–134.

KRESÁK, Ľ. 1979. Dynamical interrelations among comets and asteroids. In *Asteroids* (T. Gehrels, Ed.), pp. 289–309. Univ. of Arizona Press, Tucson.

KRESÁK, Ľ., AND M. KRESÁKOVÁ 1987. The contribution of periodic comets to the zodiacal cloud. In *Interplanetary Matter* (Z. Ceplecha and P. Pecina, Eds.), pp. 265–272. Czechosl. Acad. Sci. Publications, Prague.

LEVERRIER, U. J. 1855. Développement de la fonction qui sert de base au calcul des mouvements des planètes. *Ann. Obs. Paris* **1,** 258–342.

LEVISON, H. F., AND M. J. DUNCAN 1994. The long-term dynamical behavior of short-period comets. *Icarus* **108,** 18–36.

POINCARÉ, H. 1892. *Les Méthodes Nouvelles de la Mécanique Céleste.* Gauthier–Villars, Paris.

PORUBČAN, V., AND J. ŠTOHL 1987. The meteor complex of P/Encke. In *Interplanetary Matter* (Z. Ceplecha and P. Pecina, Eds.), pp. 167–171. Czechosl. Acad. Sci. Publications, Prague.

SOUTHWORTH, R. B., AND G. S. HAWKINS 1963. Statistics of meteor streams. *Smithson. Contrib. Astrophys.* **7,** 261–285.

STEEL, D. I. 1994. Co-genetic families amongst the planet-crossing asteroids. In *75 Years of Hirayama Asteroid Families: The Role of Collisions in the Solar System History* (Y. Kozai, R. P. Binzel, and T. Hirayama, Eds.), pp. 97–108. Astron. Soc. Pacific, San Francisco.

STEEL, D. I., D. J. ASHER, AND S. V. M. CLUBE 1991. The structure and evolution of the Taurid Complex. *Mon. Not. R. Astron. Soc.* **251,** 632–648.

ŠTOHL, J., AND V. PORUBČAN 1990. Structure of the Taurid meteor complex. In *Asteroids, Comets, Meteors III* (C.-I. Lagerqvist, H. Rickman, B. A. Lindblad, and M. Lindgren, Eds.), pp. 571–574. Uppsala Univ. Press, Uppsala.

TISSERAND, M. F. 1882. Mémoire sur les mouvements séculaires des plans des orbites des trois planètes. *Ann. Obs. Paris* **16,** E1–E57.

VALSECCHI, G. B. 1992. Close encounters, planetary masses and the evolution of cometary orbits. In *Periodic Comets* (J. A. Fernández and H. Rickman, Eds.), pp. 81–96. Universidad de la República, Montevideo.

WETHERILL, G. W. 1991. End products of cometary evolution: Cometary origin of Earth-crossing bodies of asteroidal appearance. In *Comets in the Post-Halley Era* (R. L. Newburn, Jr., M. Neugebauer, and J. Rahe, Eds.), pp. 537–556. Kluwer Academic, Dordrecht.

WHIPPLE, F. L. 1967. On maintaining the Meteoritic Complex. In *Zodiacal Light and the Interplanetary Medium,* pp. 409–426. NASA SP-150.

WHIPPLE, F. L. 1994. Presented at the meeting "Small Bodies in the Solar System and their Interactions with the Planets," 8–12 August 1994, Mariehamn, Åland, Finland.

WHIPPLE, F. L., AND S. E. HAMID 1952. On the origin of the Taurid meteor streams. *Helwan Obs. Bull.* **41,** 3–30.

WILLIAMS, J. G. 1969. *Secular Perturbations in the Solar System.* Ph.D. Dissertation, Univ. California, Los Angeles.

WISDOM, J. 1983. Chaotic behaviour and the origin of the 3/1 Kirkwood gap. *Icarus* **56,** 51–74.

WISDOM, J. 1985. Meteorites may follow a chaotic route to Earth. *Nature* **315,** 731–733.