Thermodynamics of convection in the moist atmosphere

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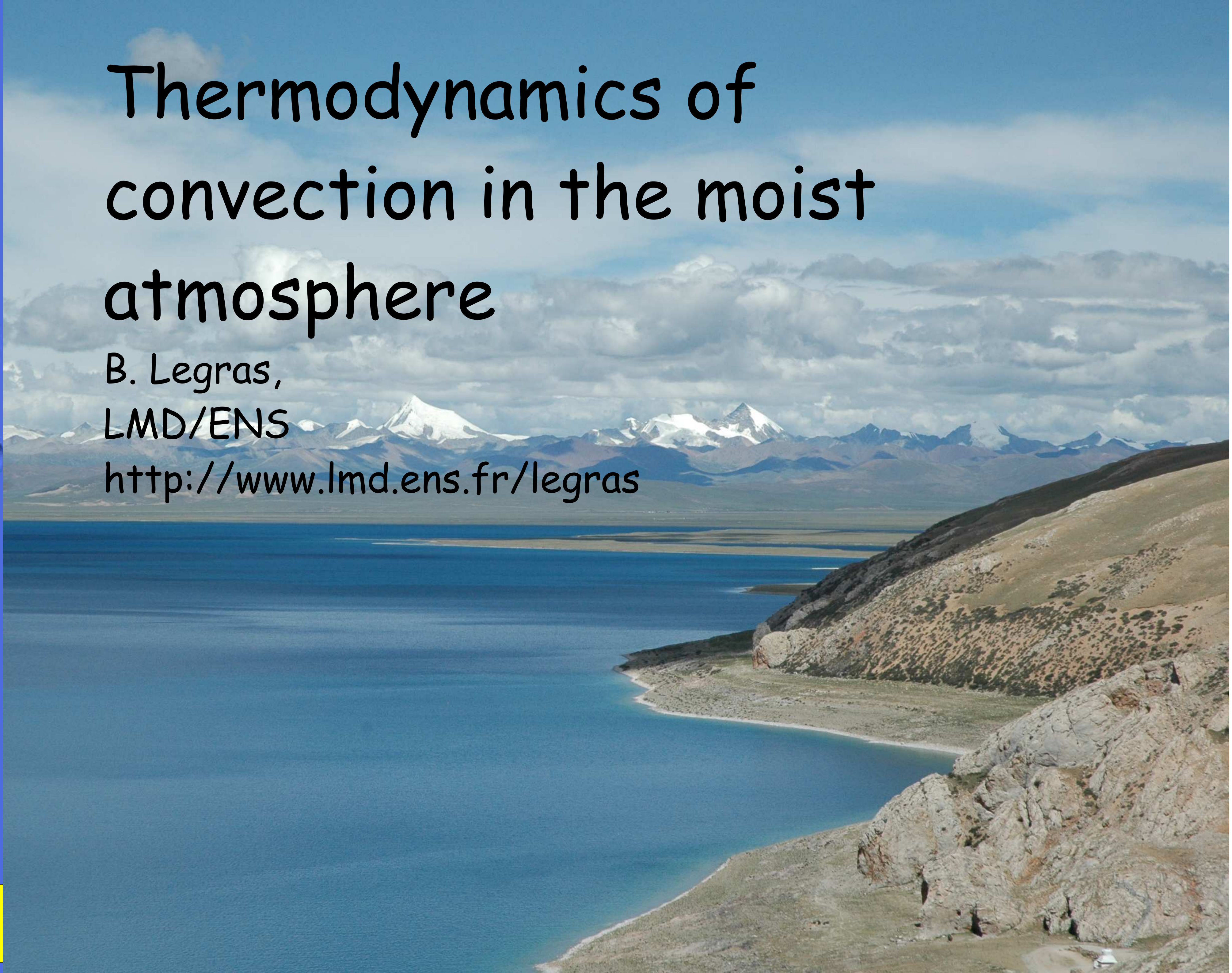


References

recommanded books:

- Fundamentals of Atmospheric Physics, M.L. Salby, Academic Press
- Cloud dynamics, R.A. Houze, Academic Press

Other more advanced books (plus avancés):

- Thermodynamics of Atmospheres and Oceans, J.A. Curry & P.J. Webster
- Atmospheric Convection, K.A. Emanuel, Oxford Univ. Press

Papers

- Bolton, The computation of equivalent potential temperature, MWR, 108, 1046-1053, 1980

OUTLINE OF FIRST PART

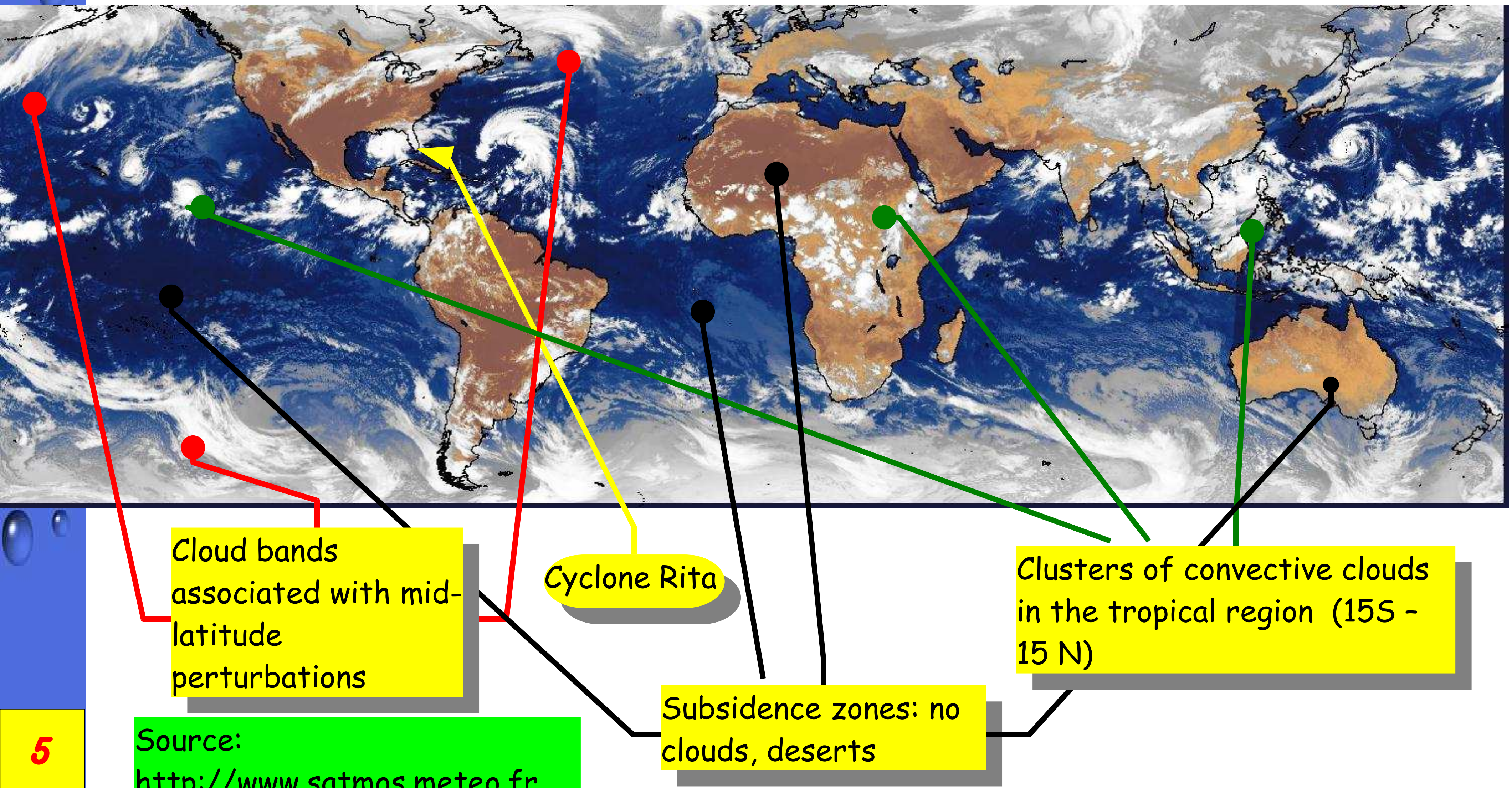
- Introduction. Distribution of clouds and atmospheric circulation
- Atmospheric stratification. Dry air thermodynamics and stability.
- Moist unsaturated thermodynamics. Virtual temperature. Boundary layer.
- Moist air thermodynamics and the generation of clouds.
- Equivalent potential temperature and potential instability.
- Pseudo-equivalent potential temperature and conditional instability
- CAPE, CIN and
- An example of large-scale cloud parameterization
-

Introduction.
Distribution of clouds and atmospheric circulation

Large-scale organisation of clouds

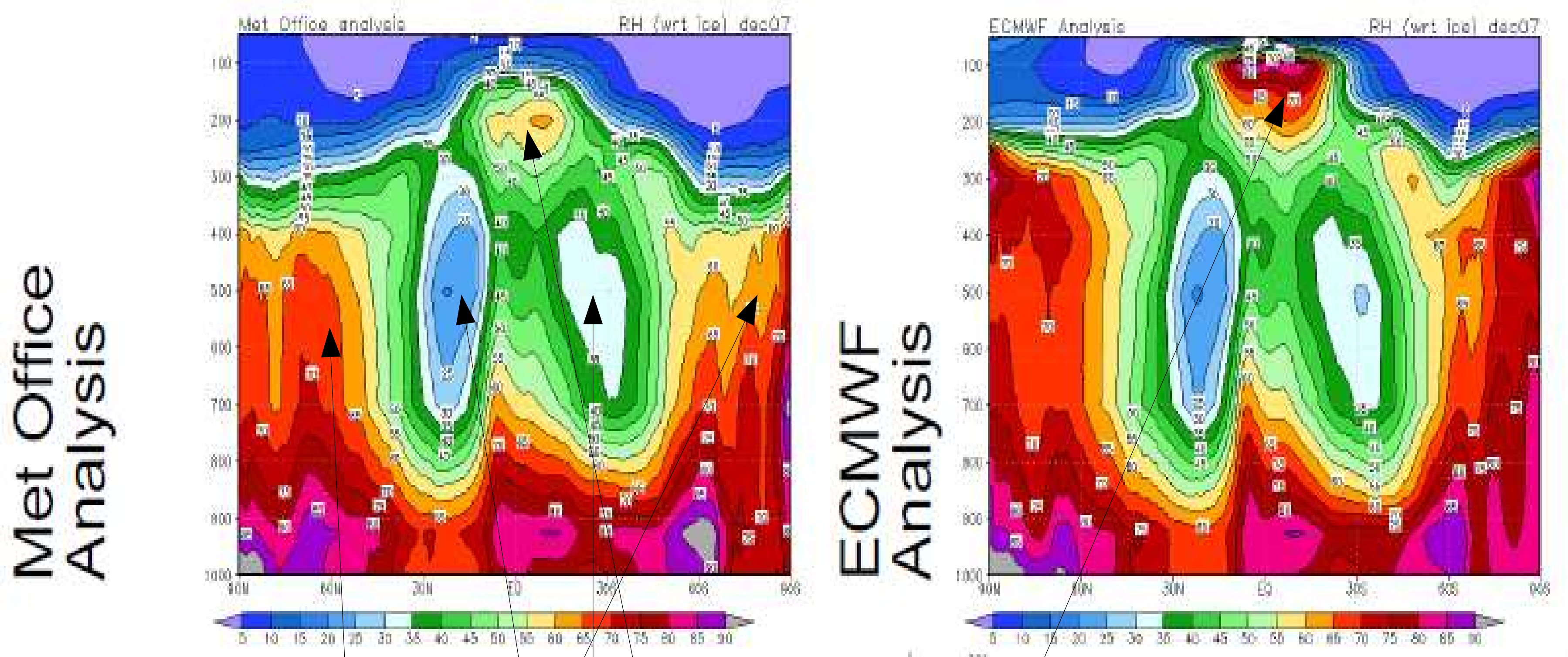
IR false color composite image, obtained par combined data from 5 Geostationary satellites   22/09/2005 18:00TU

(GOES-10 (135O), GOES-12 (75O), METEOSAT-7 (OE), METEOSAT-5 (63E), MTSAT (140E))



Source: http://www.satmos.meteo.fr

Distribution of relative humidity in the troposphere according to analysis of weather centers



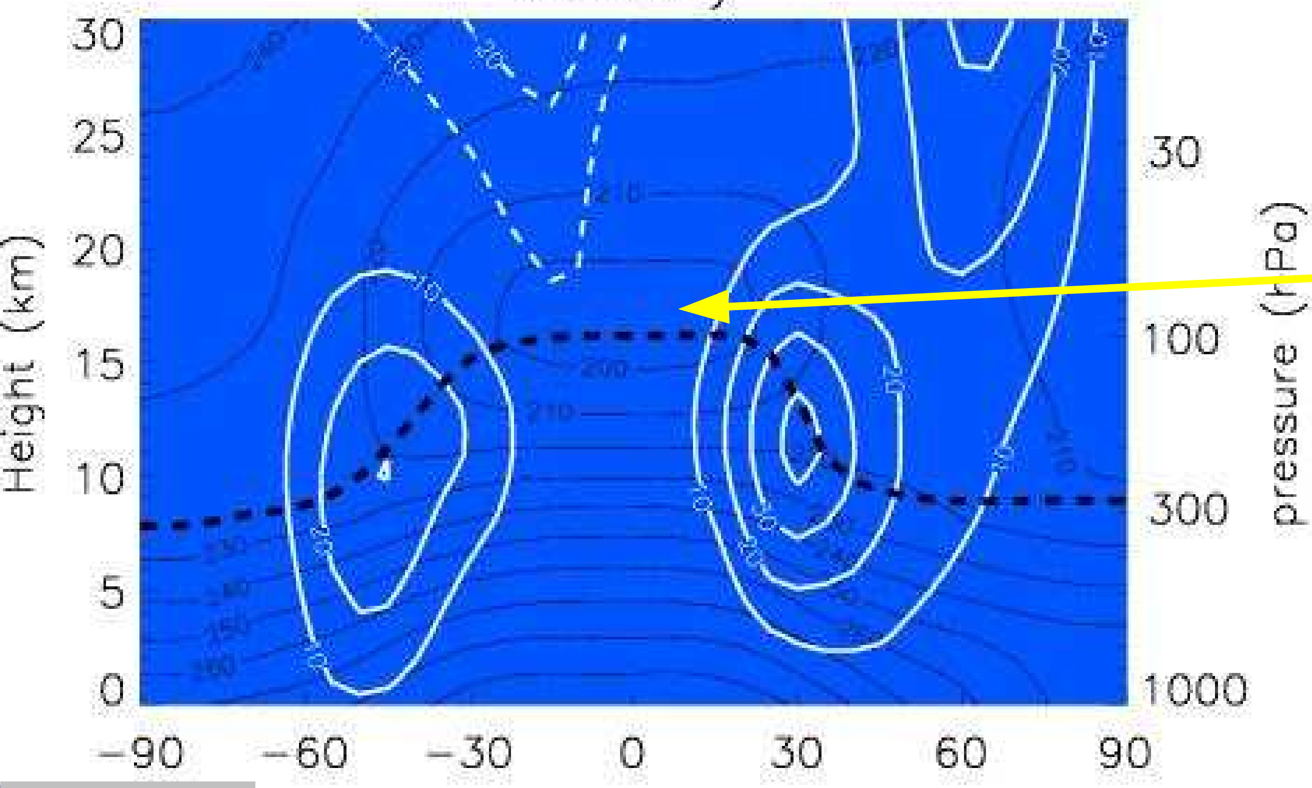
Courtesy of D. Jackson

Detrainment of tropical convection

Subsidence branch of the Hadley cell

Moistening by sloping motion associated with Rossby waves and baroclinic perturbations

Wind and temperature at the tropopause

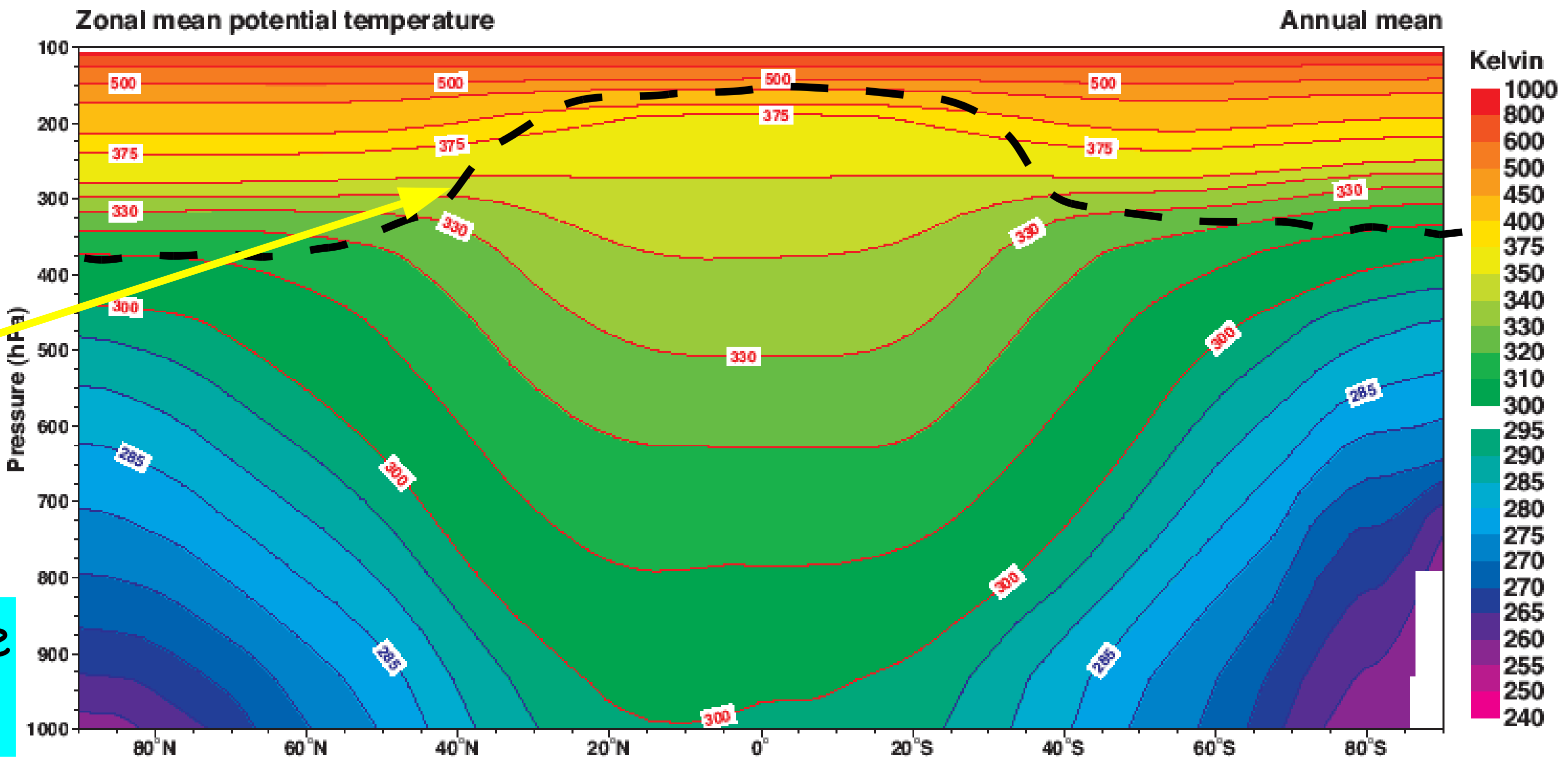


Temperature minima at the tropical tropopause

Subtropical jet winds associated with tropopause drop

B. Randell

ERA-40 Atlas, 2005

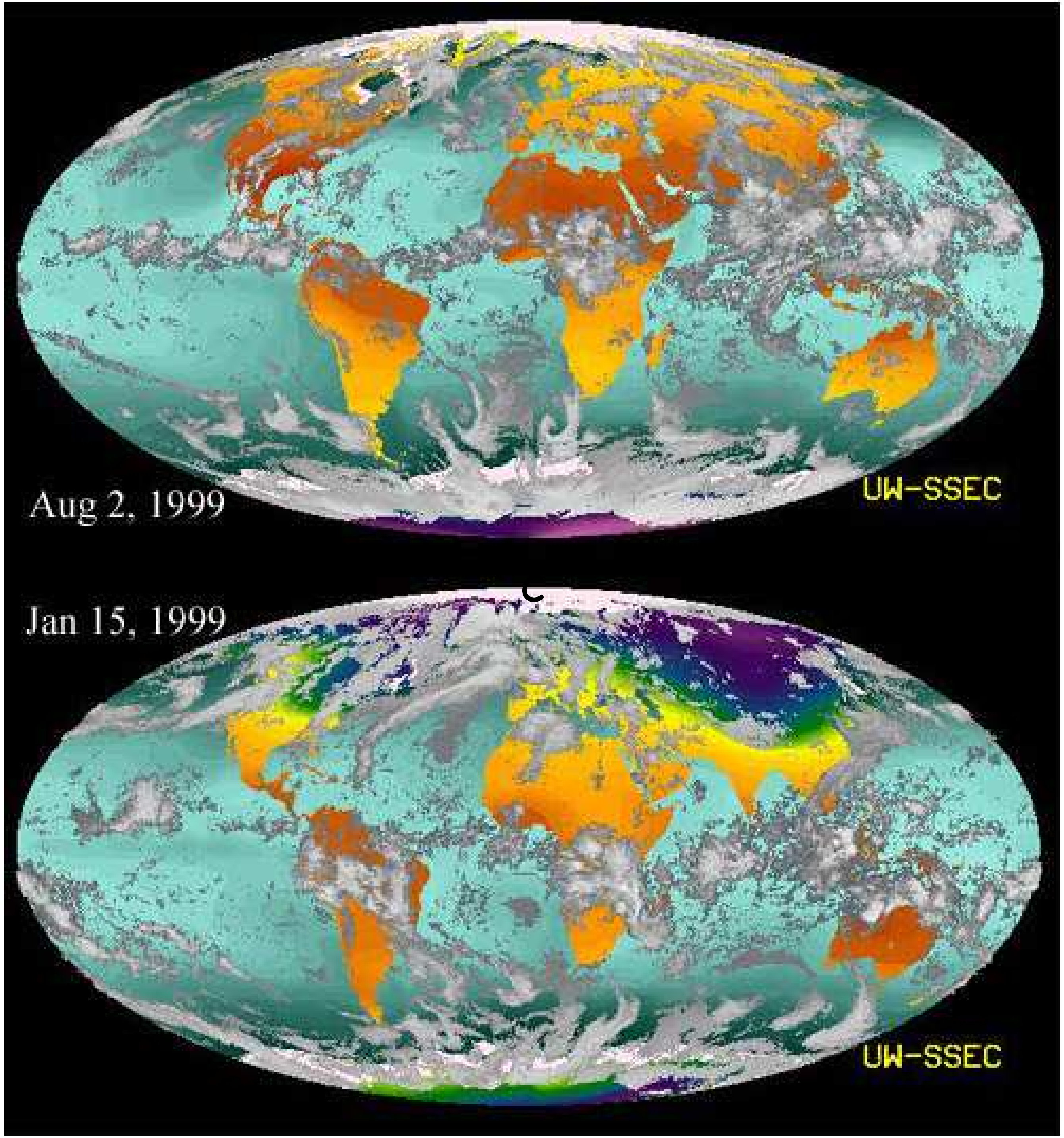


Isentropic surfaces crossing the tropopause in the subtropics

Potential temperature at the tropopause

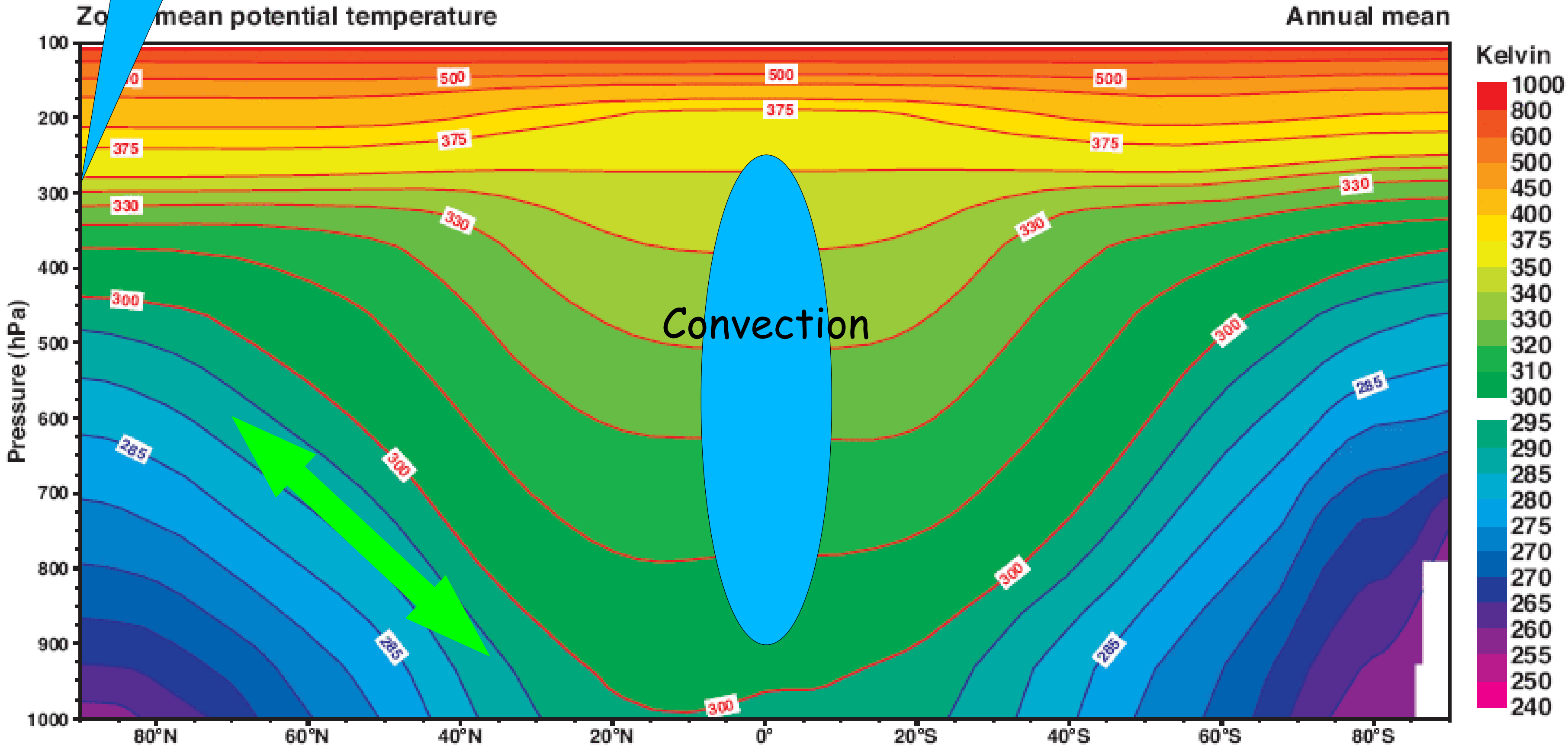
Cloud cover
ISSCP data

comparison
January-July

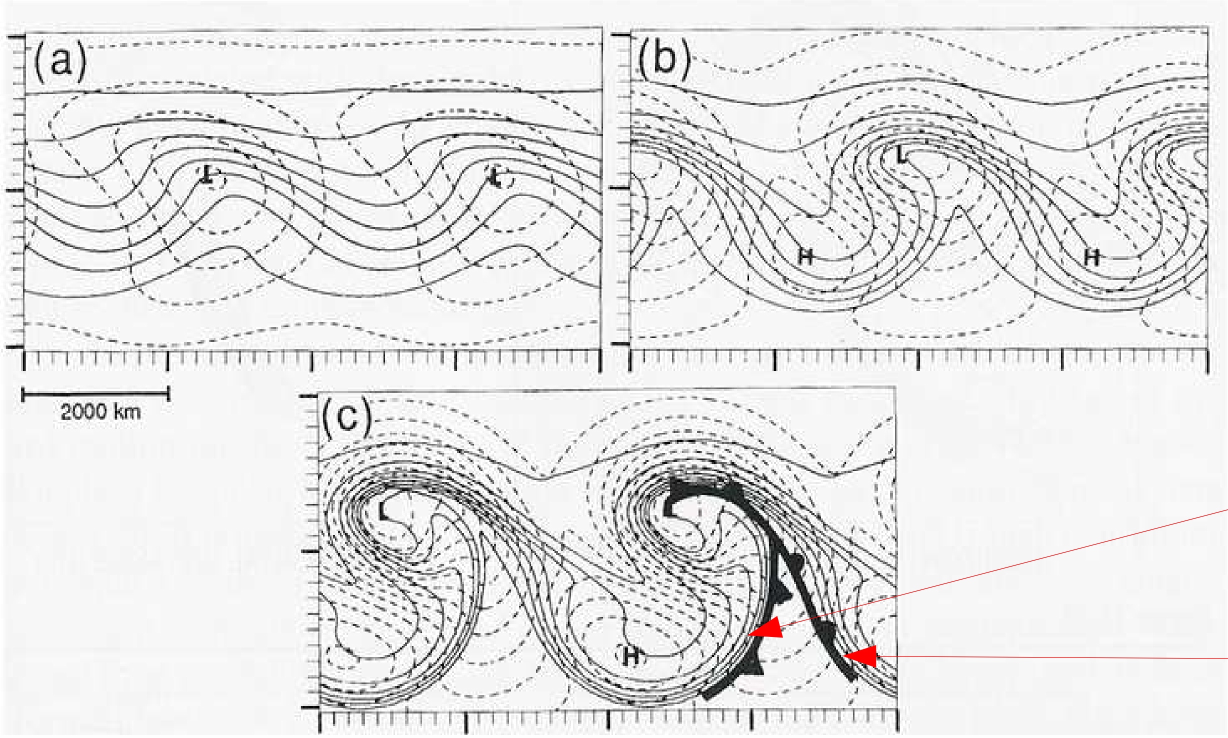


Beware of error in
ERA-40 atlas.
200 HPa should be there

ECMWF ERA-40 atlas



In the mid and high latitudes, poleward isentropic motion is accompanied by upward motion.
In the tropic, vertical upward motion needs heating.



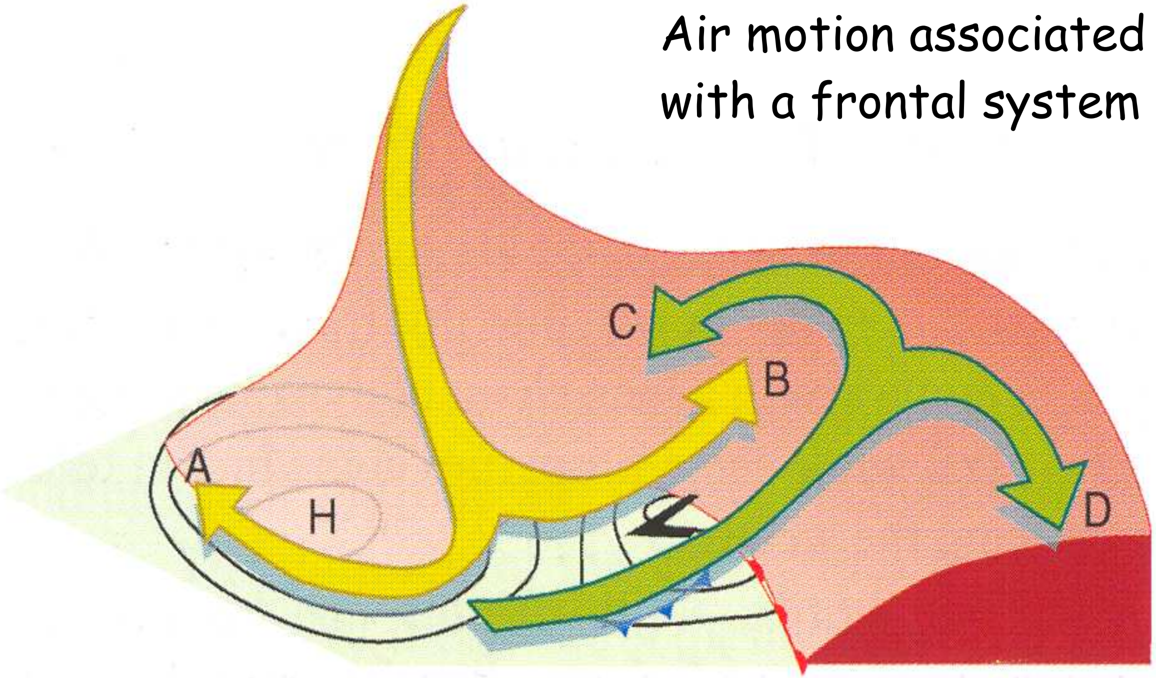
Development of an idealized baroclinic perturbation

- - - - - = pression

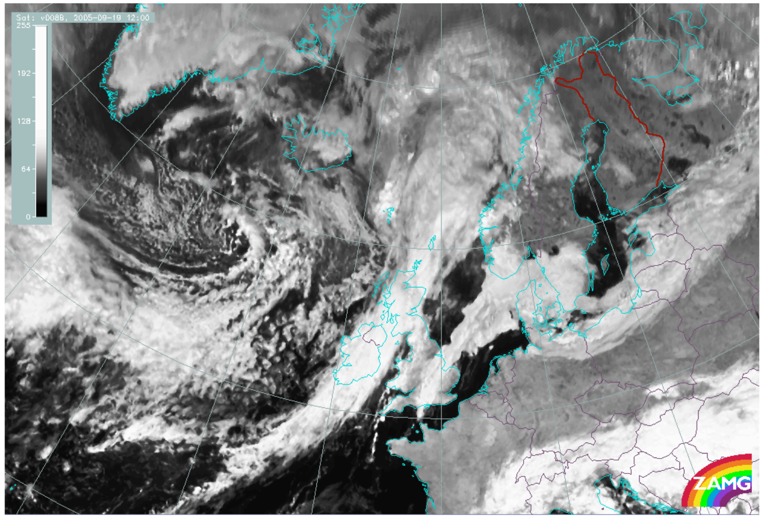
———— = température

Cold front

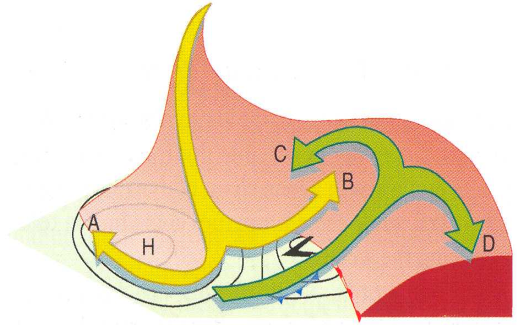
Warm front



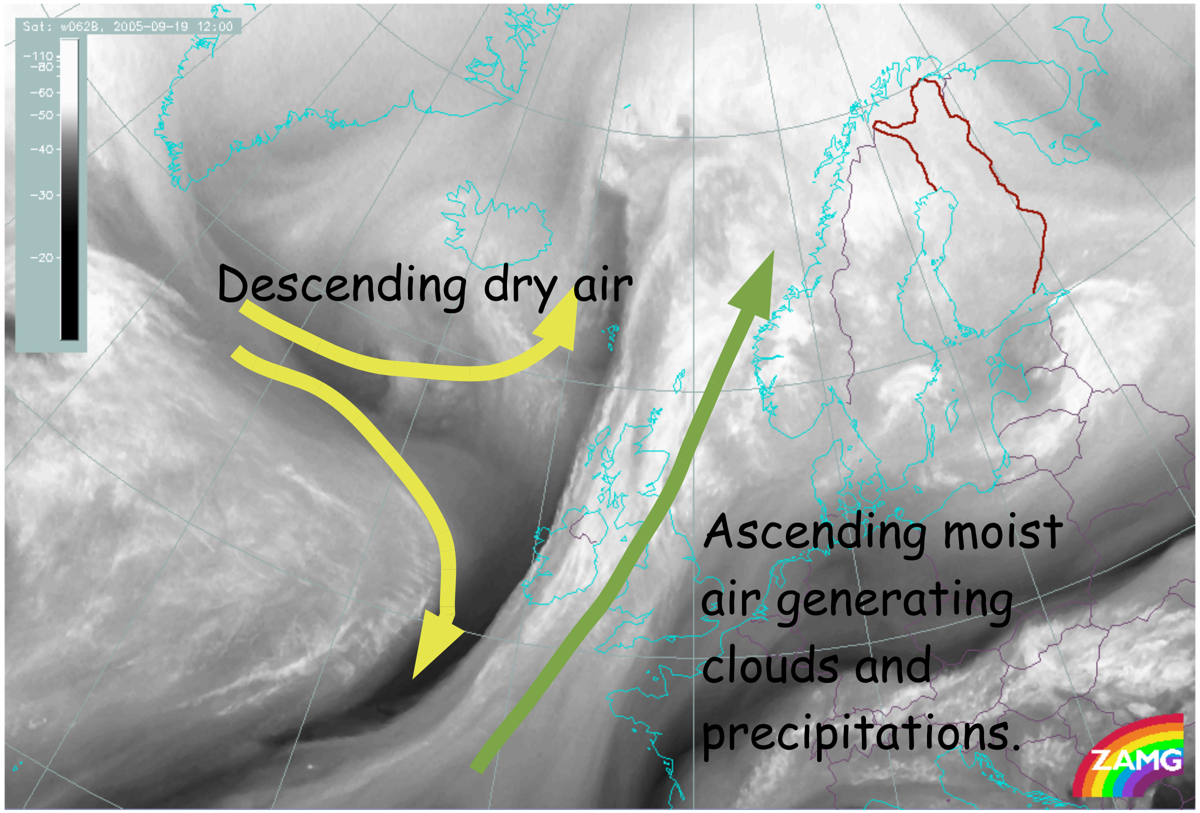
Air motion associated with a frontal system .



Visible channel
Meteosat

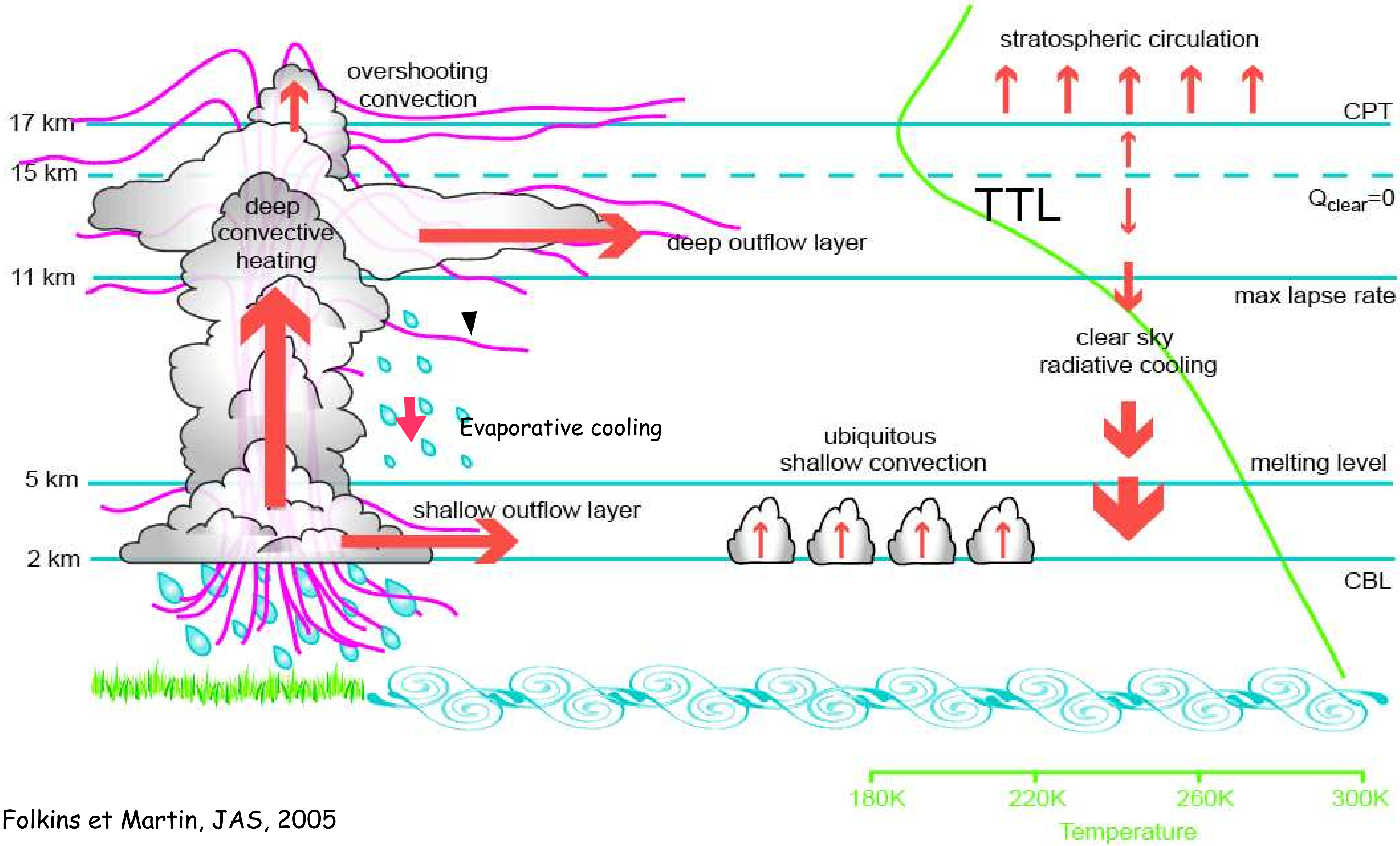


Water vapour channel Meteosat



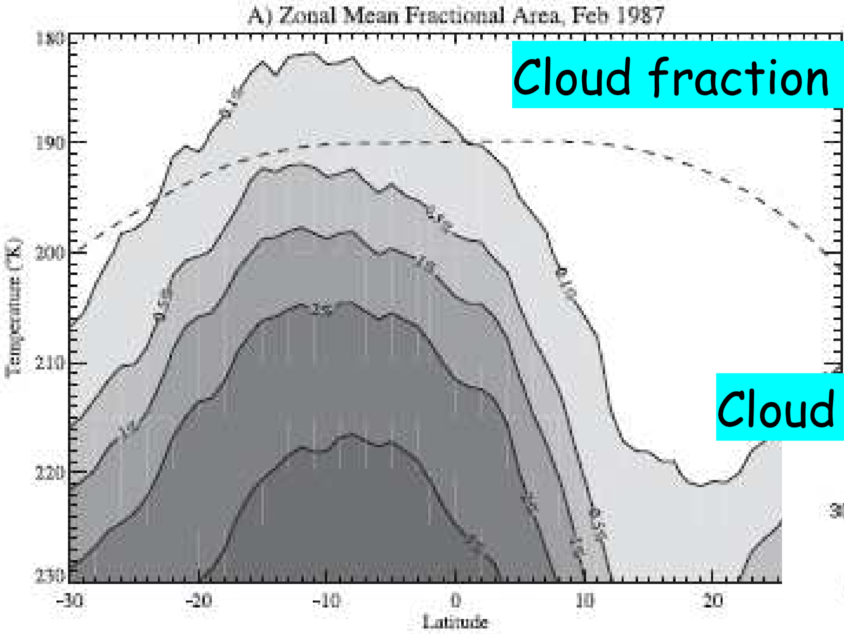
Although mixing and convective instability do occur during the development, the main ingredient is adiabatic baroclinic instability, that is isentropic motion.

Tropical Tropopause Layer and Deep Convection



Folkins et Martin, JAS, 2005

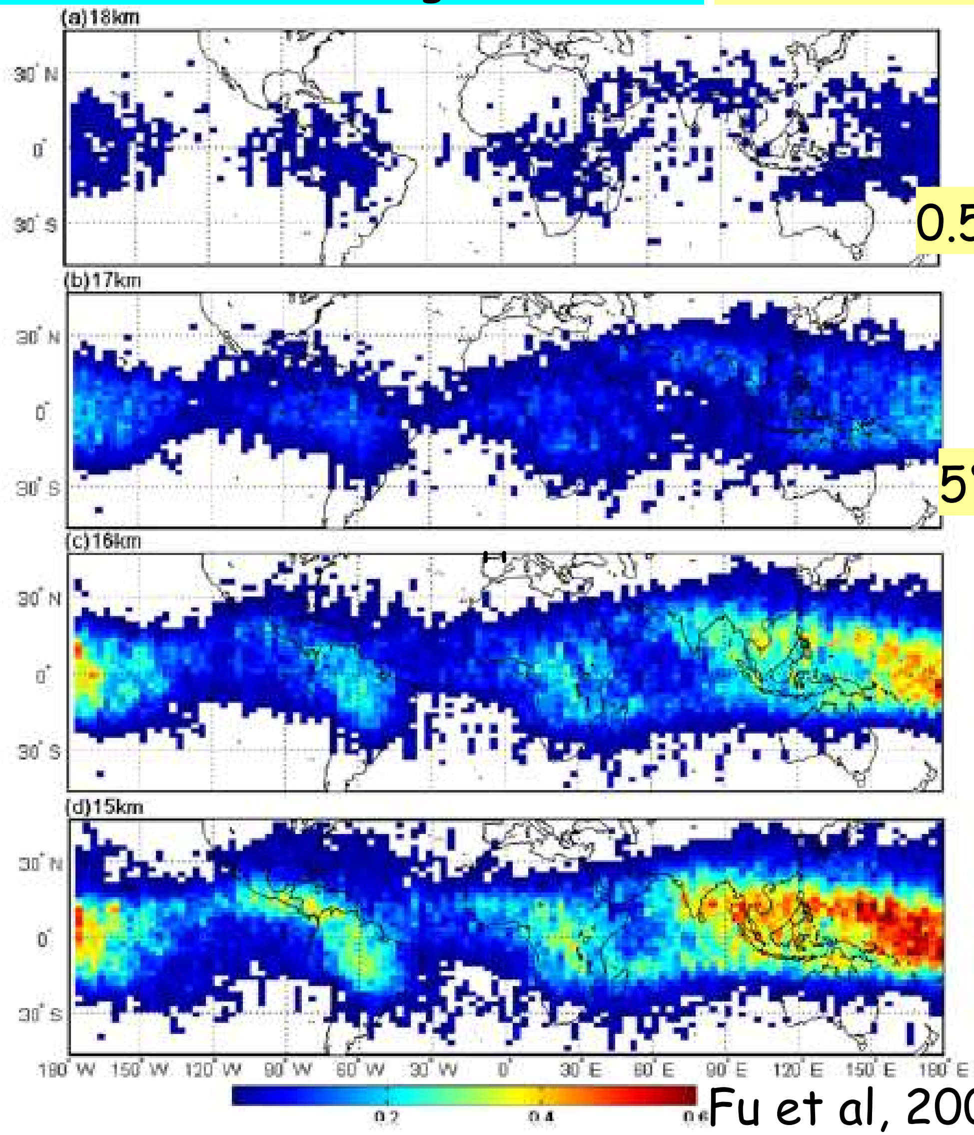
Cloud fraction according to GCI



Gettelman et al., JGR, 2002

However, convection hardly reaches the tropopause. Most clouds detrain below 15 km. Only a very small fraction (<0,5% in all seasons) is reaching the tropopause.

Cloud fraction according to CALIOP 18,5km 0.05%



0.5%

5%

Fu et al, 2007

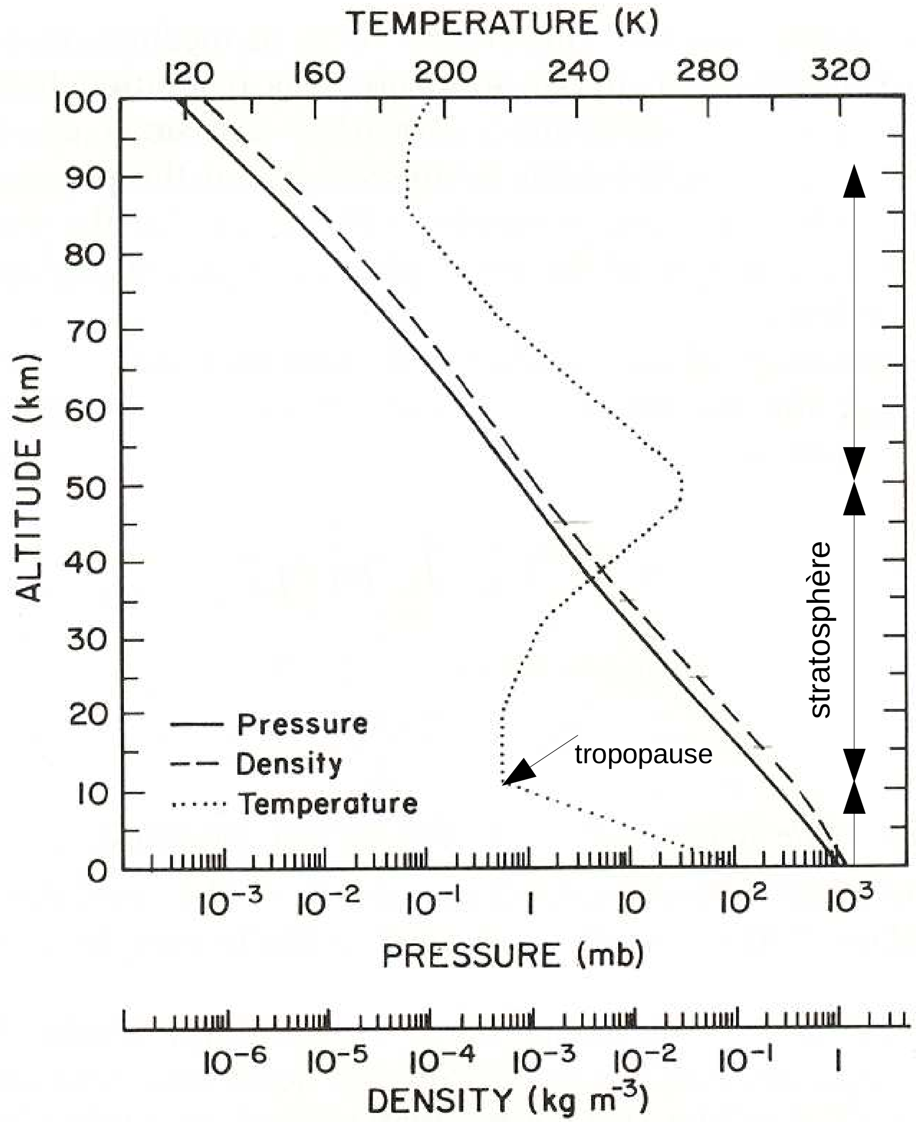
Liu and Zipser, JGR, 2005
Rosslow and Pearl, GRL, 2007
Fu et al, GRL, 2007

Atmospheric stratification
Dry air thermodynamics
and stability

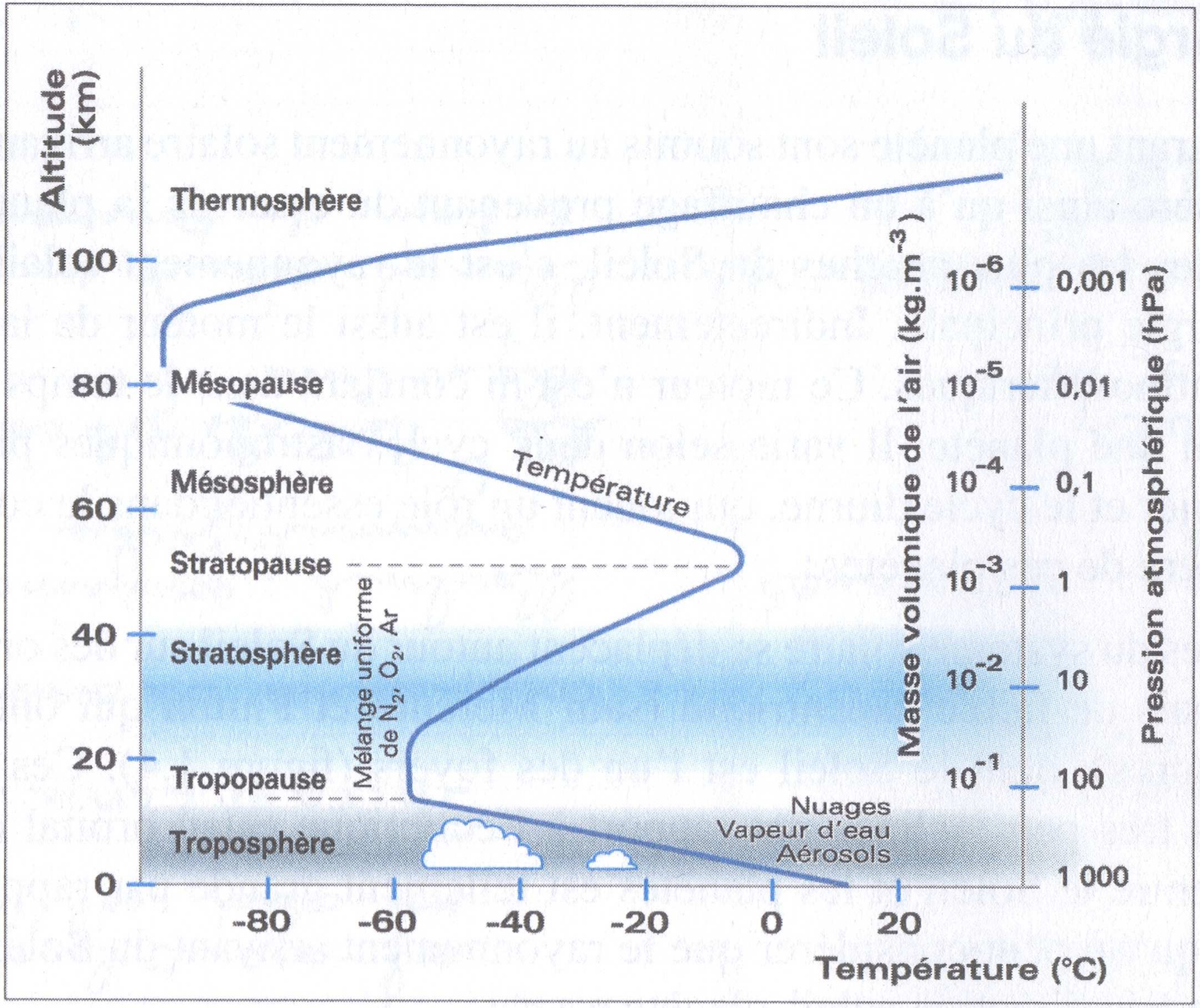
Stratification and composition

The atmosphere is stratified:

- Exponential decay of pressure and density from the ground to 100 km
- Several layers are distinguished from the temperature profile
    - Troposphere from 0 to 12 km (18 km under the tropics)
    - Stratosphere above up to 50 km
    - Mesosphere from 50 to90 km
    - 90% of the mass under 20 km
    - Standard density (à 1013 hPa et 273K): ρ=1,29 kg m^{-3}



The troposphere and the stratosphere are separated by the tropopause



Altitude (km)
Thermosphère
100
80
Mésopause
Mésosphère
Température
60
Stratopause
40
Stratosphère
Mélange uniforme de N_2, O_2, Ar
20
Tropopause
Troposphère
Nuages
Vapeur d'eau
Aérosols
0
-80
-60
-40
-20
0
20
Température (°C)
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Masse volumique de l'air (kg.m^{-3})
0,001
0,01
0,1
1
10
100
1 000
Pression atmosphérique (hPa)

Composition of the atmosphere*

Nitrogen N_2	0,7808	homogeneous
Oxygen O_2	0,2095	homogeneous
Water H_2O	<0,030	highly variable
Argon A	0,0093	homogeneous
CO_2	385 ppmv	homogeneous (quasi)
Ozone O_3	10 ppmv	stratosphere
Methane CH_4	1,6 ppmv	decay with z
Nitrous oxide N_2O	350 ppbv	decay with z
CO	70 ppbv	
NO, CFC-11, CFC-12	< 0,3 ppbv	

Mean molar mass M=28,96 g

*: composition is given as volume mixing ratio

Complementary note: Definition of mixing ratios

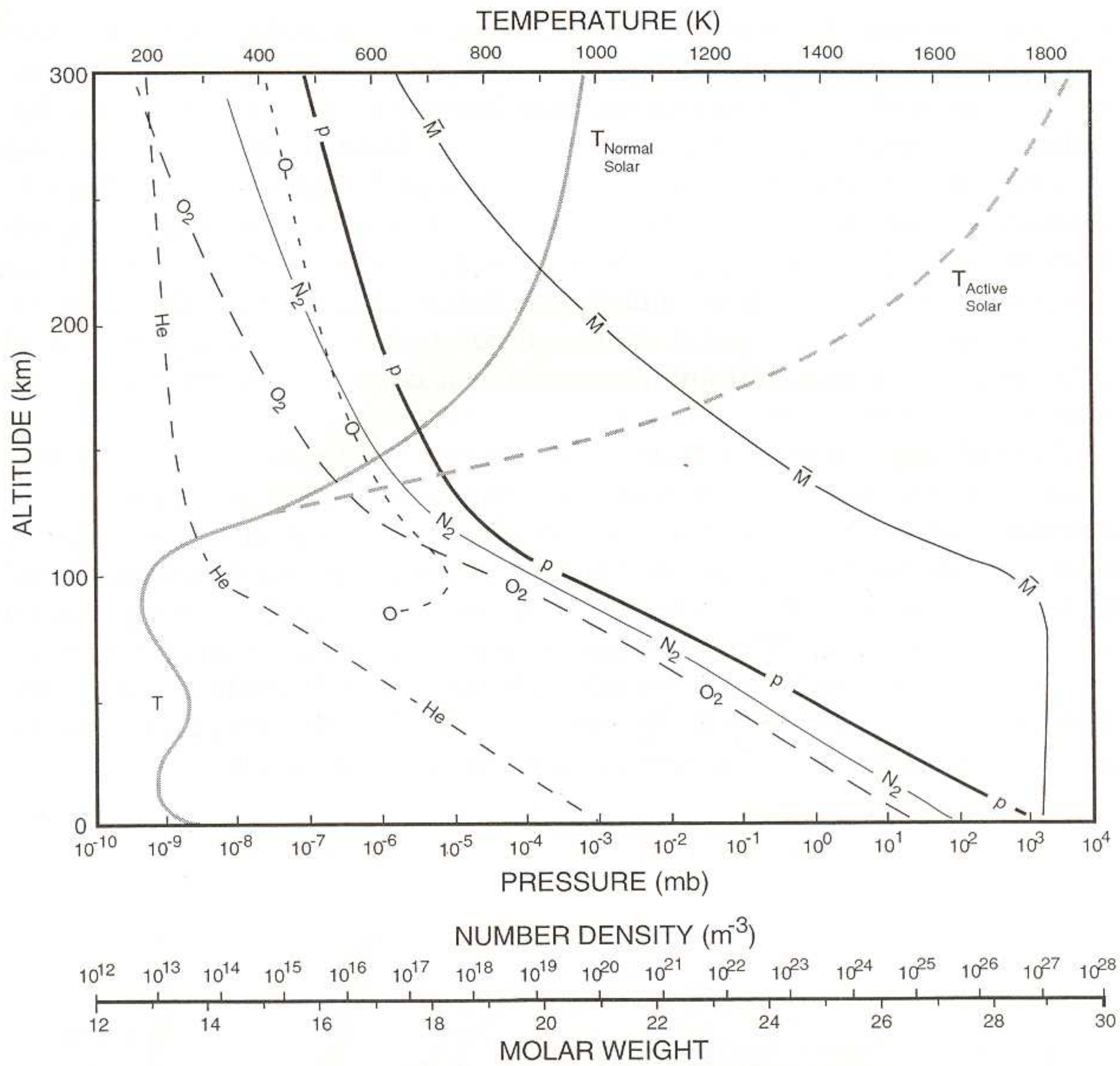
The proportions of gases in the air are given as mass or volume mixing ratios. The reference is dry air (without water) with molar mass M_d=28,96 g and density ρ_d. For a minor gas with molar mass M and density ρ, the mass mixing ratio is $r_m = \rho/\rho_d$. We may also use the volume mixing ratio defined as the ratio between the gas partial pressure p to the dry air pressure p_d, $r_v = p/p_d$. The relation between the two ratios is given by $r_v = r_m M_d/M$. The volume mixing ratio indicates the proportion of minor gas as a number of molecules per molecule of dry air. When this proportion is very low, multiplicative factors are used and the mixing ratio is given in ppmv (part per million in volume ⇔ factor 10^6), ppbv (part per billion in volume ⇔ factor 10^9) or pptv (part per billion in volume ⇔ factor 10^{12}).

Water only, which can reach r_v=0.03 in tropical regions, is able to change significantly the air density. Other gases are in too small proportion to affect the density.

Traditionally, the mass mixing ratio is used for the thermodynamic properties of moist air. Volume mixing ratios are mostly used in chemistry and transport studies.

The composition of the atmosphere in major components (N_2, O_2) varies little until 100 km.

There are , however, strong variations among minor components (H_2O, O_3, ...)



Thermodynamics of dry air

Ideal gas law $p = \rho RT$ where $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$

- Ideal gas enthalpy $H = C_p T$ where $C_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$, heat capacity per unit mass at constant pressure. H depends only of the temperature

- At constant pressure, for quasi-static transform:

$\delta Q = C_p\, dT = dH = T\, dS$ (S: entropy)

- More generally

$\delta Q = T\, dS = dU + p\, d(1/\rho) - 1/\rho\, dp$

$= C_p\, dT - 1/\rho\, dp = C_p\, dT - RT/p\, dp = C_p\, (T/\theta)\, d\theta$

where θ is the potential temperature

$$\theta = T(p_0/p)^{\kappa} \text{ avec } \kappa = R/C_p = 2/7$$

Hydrostatic law and stratification

In the vertical, the air is essentially in hydrostatic equilibrium: by averaging over aan horizontal area of a few km^2, verical speed is of the order of a few cm/s and vertical acceleration is negligeable in front of gravity

- Hydrostatic law $dp/dz + \rho g = 0$

Combining with ideal gas law, we obtain $dp/p = -g/RT\ dz$ and, for a uniform temperature (gross simplification, valid within 20%) $T_0 = 255$ K , we obtain

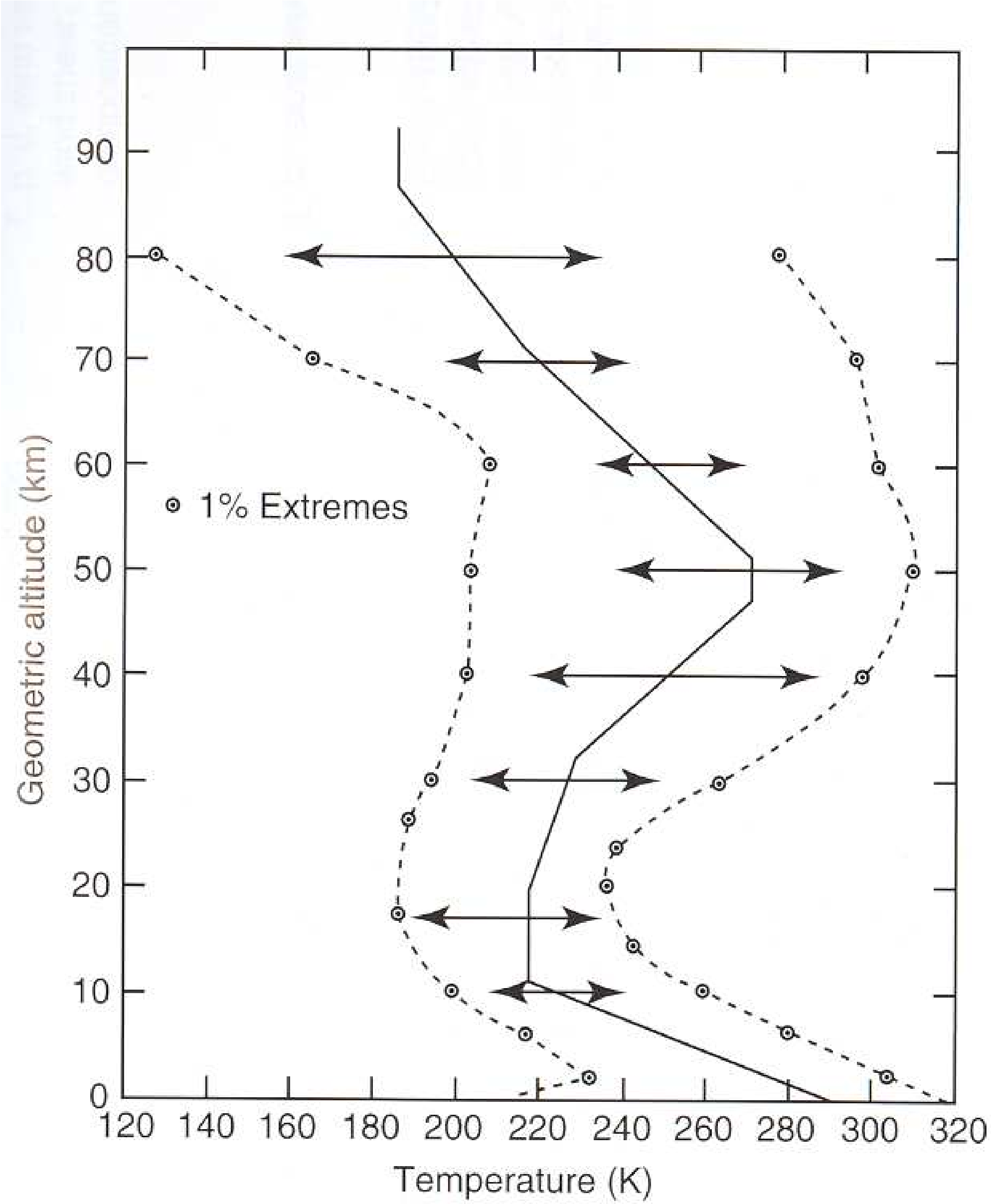
$$p = p_0 \exp(-z/H)$$

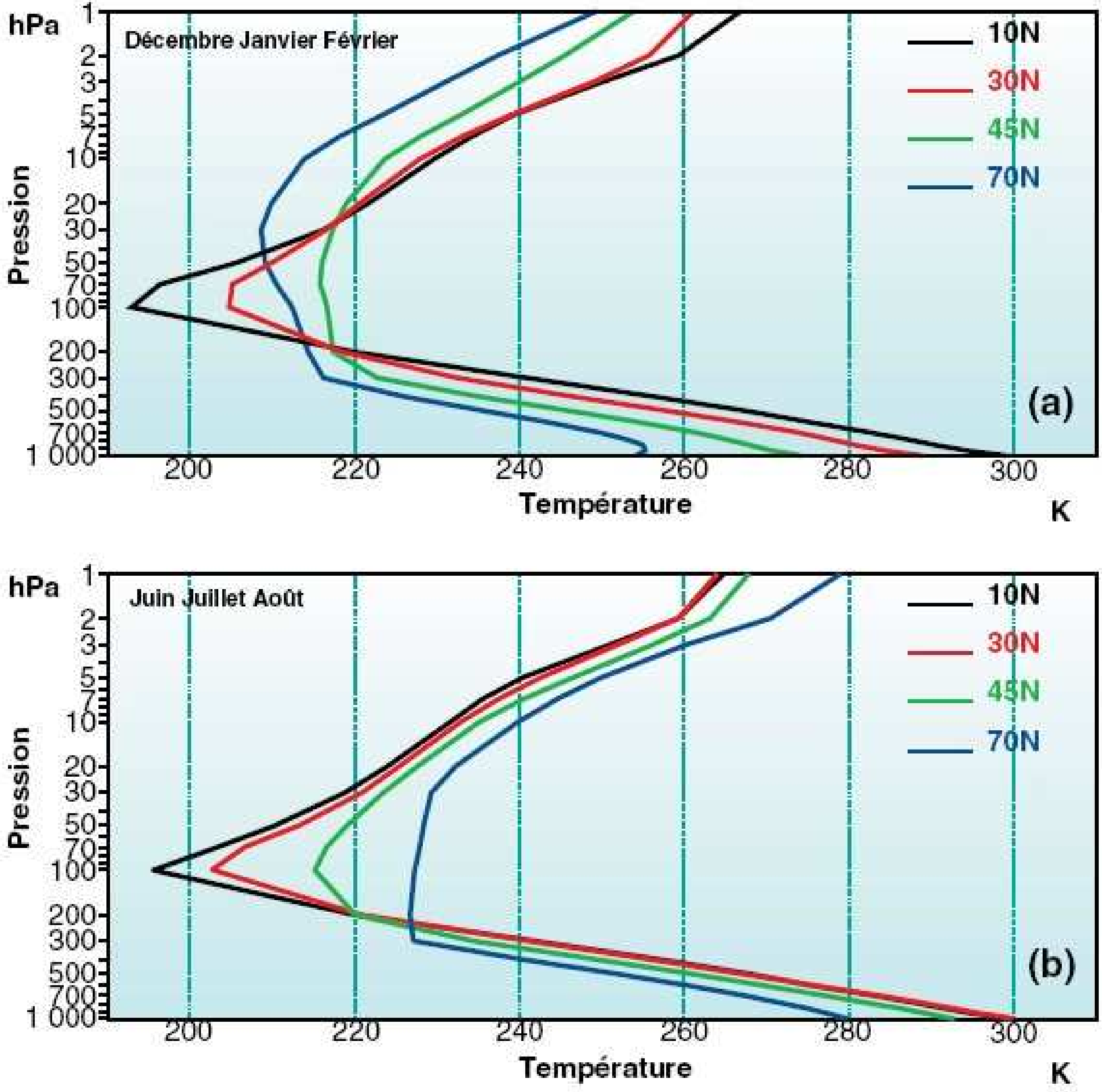
with $H = RT_0/g \approx 7{,}4$ km, hight scale.

Pressure decreases by half every 5 km (because $H \ln(2) \approx 5$ km)

The vertical profile of temperature cannot be explained with simple laws. It depends on vertical mixing and heat transport and on the radiative emissions and absorption.

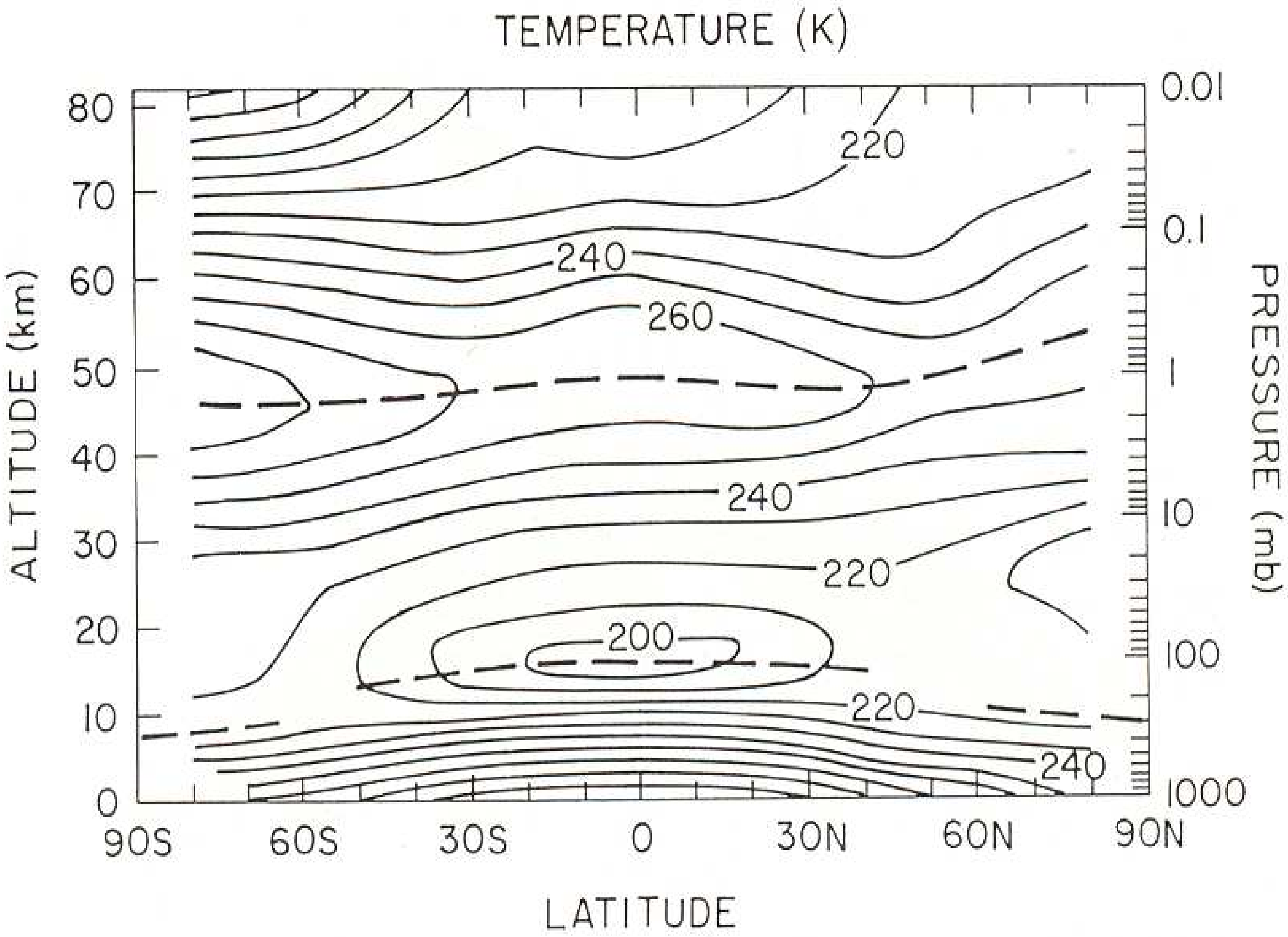
- At the ground, variations of 100 K but 50 K on the average between pole and equator
- Temperatures in the tropopause region are less varying but reach very small values. Very low temperatures (190 K) at the tropical tropopause and in the winter polar lower stratosphere.





hPa
Décembre Janvier Février
10N
30N
45N
70N
(a)
Pression
1
2
3
5
7
10
20
30
50
70
100
200
300
500
700
1 000
200
220
240
260
280
300
Température
K
hPa
Juin Juillet Août
10N
30N
45N
70N
(b)
Pression
Température
K

Meridional distribution of the mean temperature



To retain

• The atmosphere until 100 km is composed of nitrogen (78%) , oxygen (21%) and argon (0,9%)in fixed quantity to which are added minor gases. The most important is water which is in highly variable concentration (from 35g/kg at the tropical in tropical regions to a few mg/kg at 100 hpa in the tropics and above in the stratosphere).

• The atmospheric ozone is concentrated at 90% in the stratosphere. It filters solar UV rays (under 290nm) and converts this energy to heat, maintaining a growing temperature profile from the tropopause to 50 km.

• The atmosphere is stratified. It is in hydrostatic equilibrium for motion averaged over horizontal domains larger than 10x10 km. Pressure and density decrease exponentially with altitude, being divided per 2 every 5 km

• Air thermodynamics is well described by the ideal gas law.

•

• The temperature decreases by about 6,5 K par km in the troposphere until the tropopause. Beyond this frontier, the temperature grows again until about 55 km. The height of the tropopause is variable with latitude. It is 6 km in polar regions, 12 km (300 hPa) at mid-latitudes and 18 km (100 hPa) in th tropical zone near the equator. The tropical tropopause temperature is very low of the order of 200K.

Dry air thermodynamics (cont'd)

Ideal gas law $p = \rho RT$ where $R = 287\ J\ kg^{-1}\ K^{-1}$

Ideal gas enthalpy $H = C_p T$ where $C_p = 1005\ J\ kg^{-1}\ K^{-1}$, heat capacity per unit mass at constant pressure. H depends only of the temperature

At constant pressure, for quasi-static transform:

$\delta Q = C_p\, dT = dH = T\, dS$ (S: entropy)

More generally

$\delta Q = T\, dS = dU + p\, d(1/\rho) - 1/\rho\, dp$

$= C_p\, dT - 1/\rho\, dp = C_p\, dT - RT/p\, dp = C_p\, (T/\theta)\, d\theta$

where θ is the potential temperature

$\theta = T(p_0/p)^{\kappa}$ avec $\kappa = R/C_p$ (=2/7 for diatomic gas)

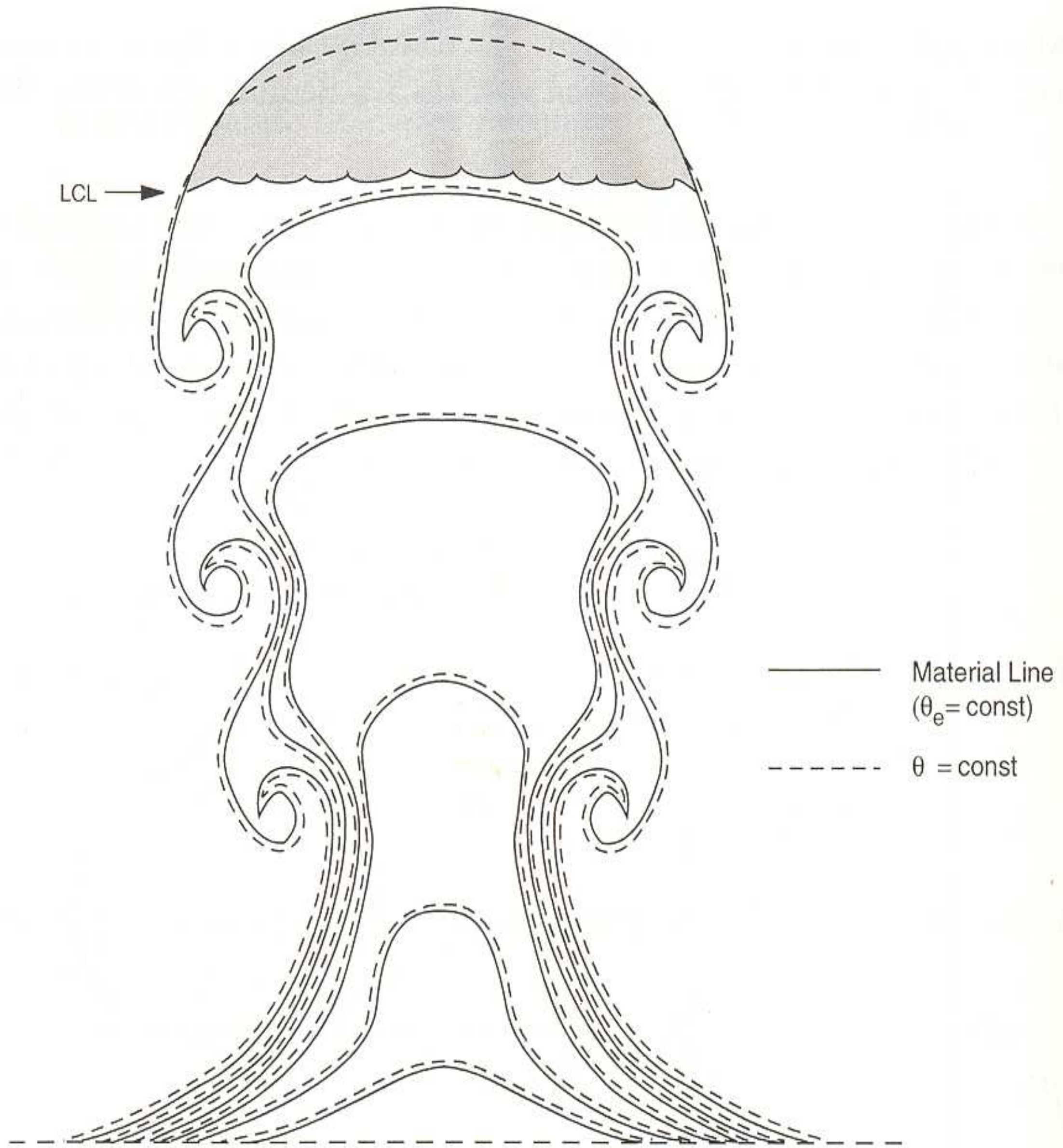
Potential temperature

Convective motion is on the first approximation adiabatic and reversible (heat exchanges much slower than pressure equilibration and weak mixing).

Conservation of potential temperature

$\theta = T(p_0/p)^{\kappa}$

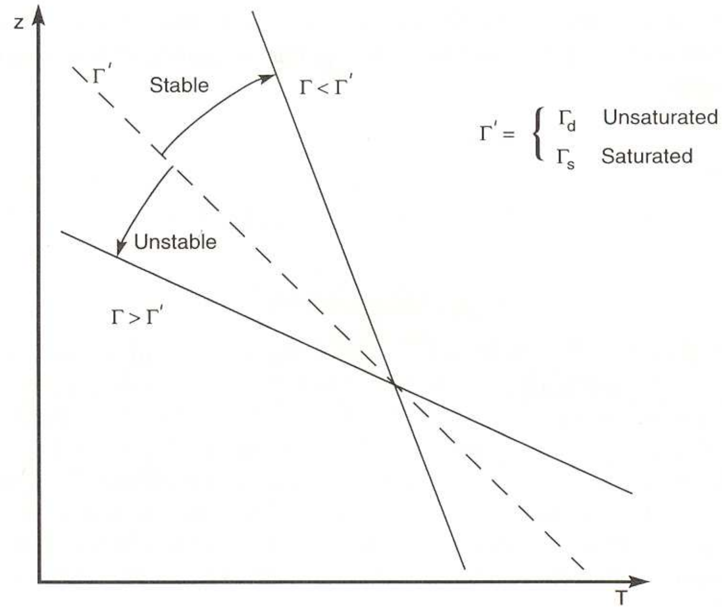
with $\kappa = R/C_p$



Dry air convective intability as a function of the atmospheric profile

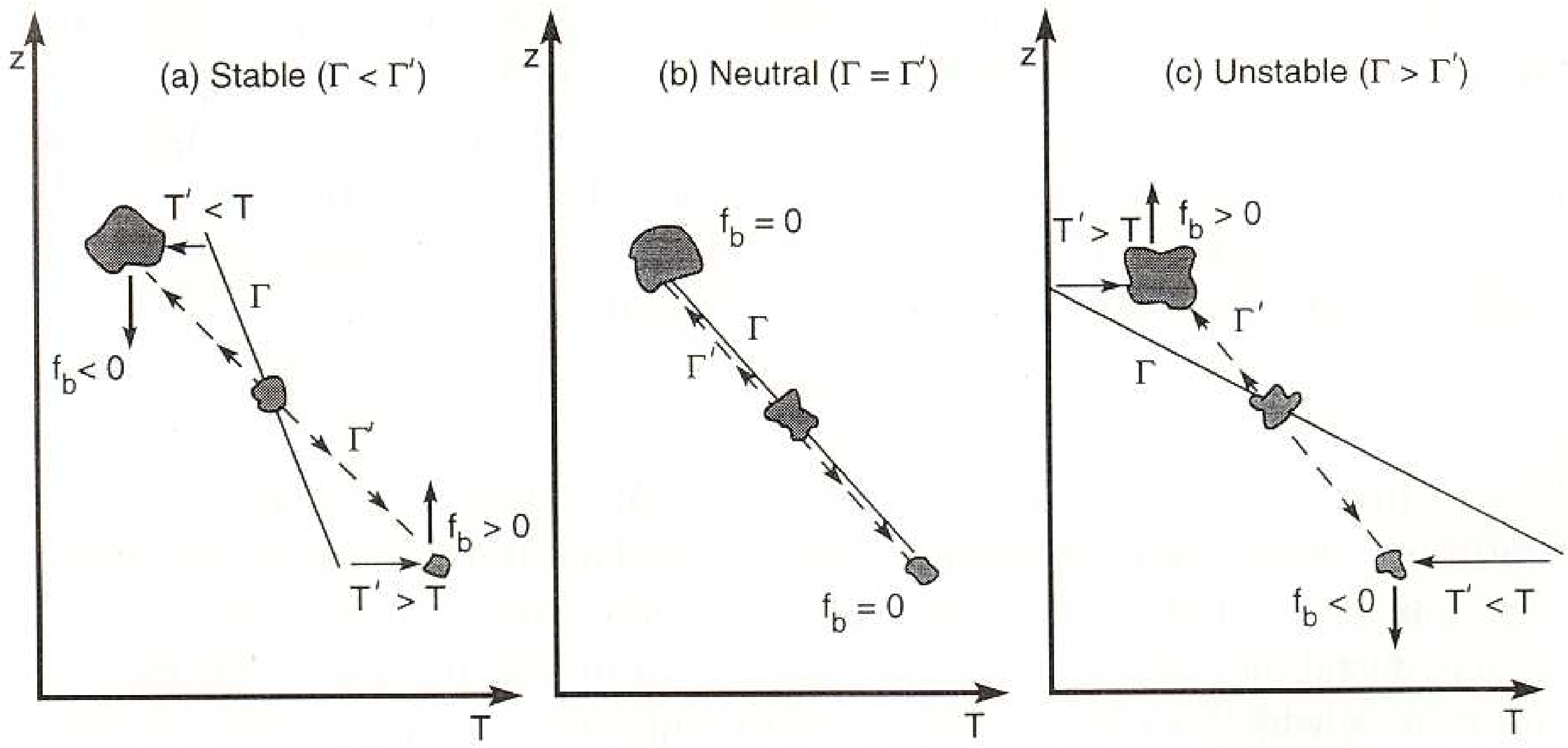
Temperature gradient Γ = - dT/dz

Adiabatic gradient   Γ' = g/Cp



$$We\ use\ 0 = \delta Q = C_p dT - \frac{1}{\rho} dp\ and\ \frac{dp}{dz} + \rho g = 0$$

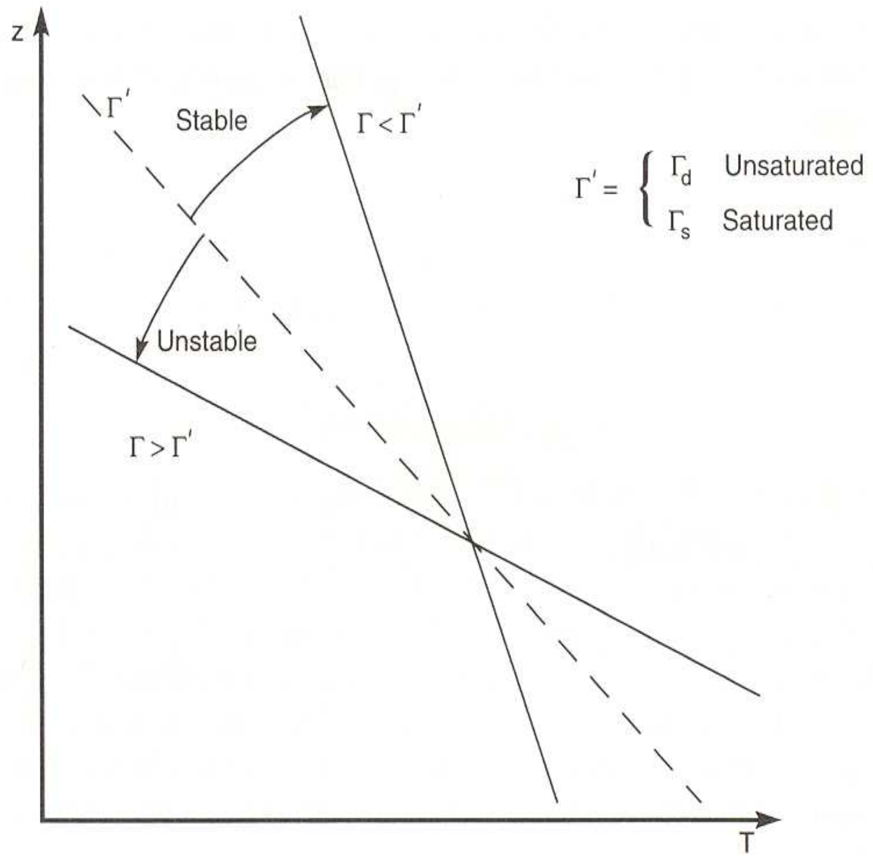
$$to\ obtain\ \frac{dT}{dz} = \frac{-g}{C_p}\ during\ adiabatic\ motion$$

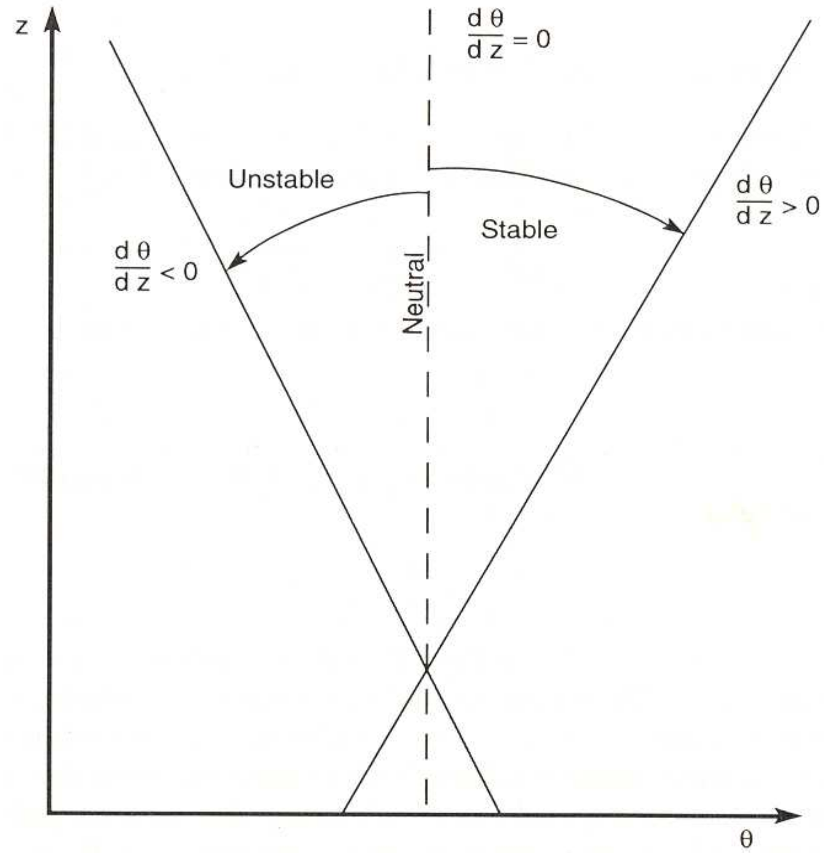


We assume that vertical motion of the parcel is along an adiabatic transform and we compare the new temperature T' to the local environment temperature to determine whether the air is stable or unstable.

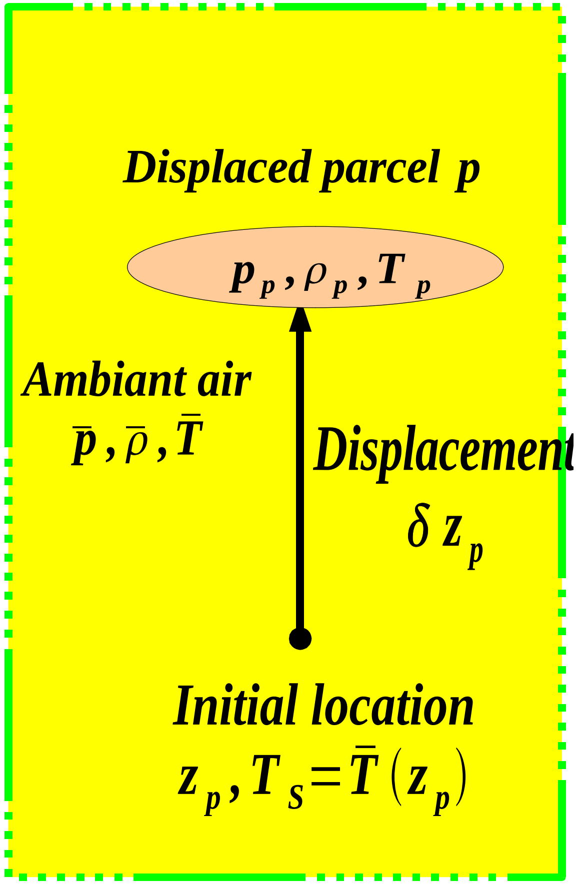
Instability in terms of $\Gamma = dT/dz$ and of the potential temperature θ

Criterion $\Gamma > \Gamma'$ or $d\theta/dz < 0$





Vertical oscillation motion of air parcel



Notations:

- Thermodynamic properties of the ambiant air : $\bar{p}, \bar{\rho}, \bar{T}$
- Thermodynamic properties of the parcel : p_p, ρ_p, T_p

The ambiant air is in hydrostatic equilibrium: $\frac{\partial \bar{p}}{\partial z} + \bar{\rho} g = 0 \qquad (1)$

The pressure of the parcel equilibrates instantly with ambiant pressure : $p_p = \bar{p}$

The equation of adiabatic motion of the parcel is $\rho_p \frac{d^2 \delta z_p}{dt^2} + \rho_p g + \frac{\partial \bar{p}}{\partial z} = 0 \qquad (2)$

Combining (1) et (2), we obtain $\frac{d^2 \delta z_p}{dt^2} = g \frac{\bar{\rho} - \rho_p}{\rho_p} = g \frac{T_p - \bar{T}}{\bar{T}} =$ buoyancy acceleration,

the second relation being obtained by applying ideal gas law.

If T_S is the temperature of the ambiant air and the parcel at its initial location, the temperatures of the ambiant air and the parcel at the displaced location are δz_p perturbations of T_S, that is $\bar{T}=T_S+\delta\bar{T}$ and $T_p=T_S+\delta T_p$, hence $T_p-\bar{T}=\delta T_p-\delta\bar{T}$.

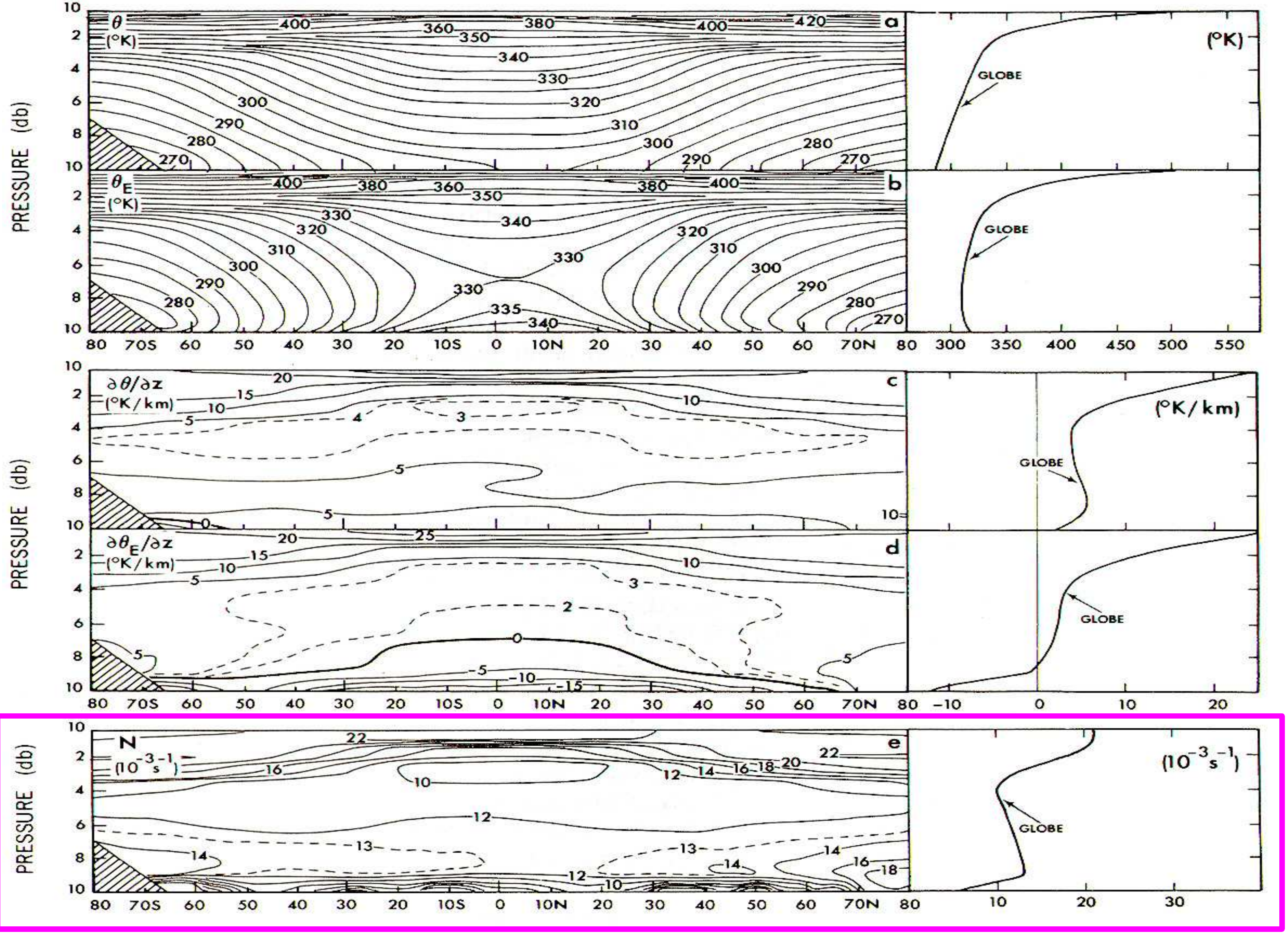
The potential temperature being defined by $\theta=T\left(\frac{p_0}{p}\right)^{\kappa}$, we have $\frac{\delta\bar{\theta}}{\bar{\theta}}=\frac{\delta\bar{T}}{\bar{T}}-\kappa\frac{\delta\bar{p}}{\bar{p}}$ for the ambiant air and $0=\frac{\delta T_p}{\bar{T}}-\kappa\frac{\delta\bar{p}}{\bar{p}}$ for the parcel moving adiabtically,

hence $\frac{d^2\delta z_p}{dt^2}=-g\frac{\delta\bar{\theta}}{\bar{\theta}}=-N^2\delta z_p$ with $N^2=\frac{g}{\bar{\theta}}\left(\frac{\partial\bar{\theta}}{\partial z}\right)$.

N is denoted as Brünt -Vaissala frequency

For $\frac{\partial\bar{\theta}}{\partial z}>0$, the Brünt-Vaissala is real and the parcel oscillates vertically with this pulsation.

For $\frac{\partial\bar{\theta}}{\partial z}<0$, the atmosphere is unstable and convective motion is established to mix the air and return to a neutral profile.



In the troposphere, N is of the order of $10^{-2}\ s^{-1}$ (period is about 10 minutes) and two times larger in the stratosphere.

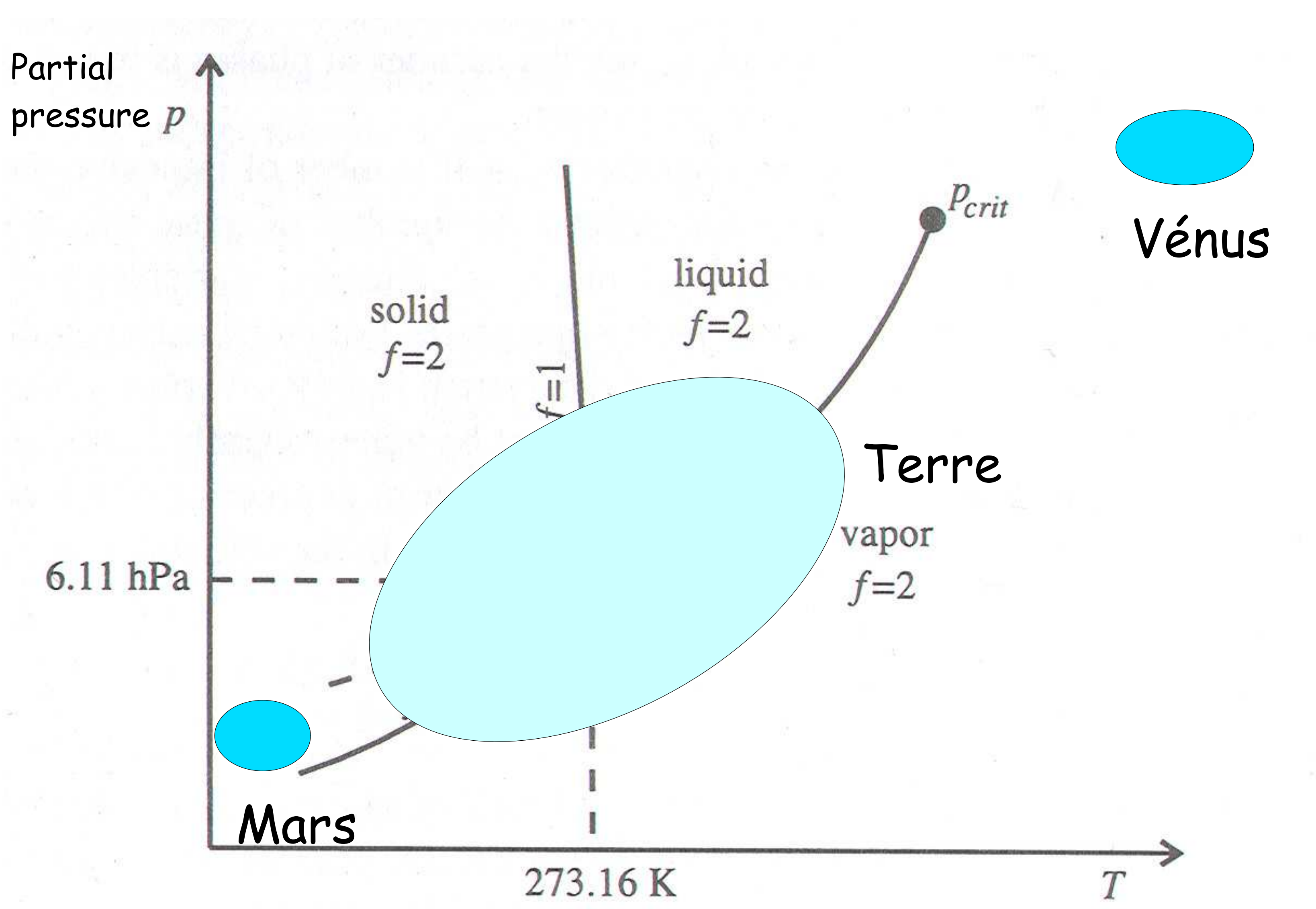
TO RETAIN

- Vertical atmospheric motion is in the first approximation adiabatic on time scales comparable with that of convection, allowing to introduce the potential temperature as an important thermodynamic variable.

- The vertical stability of the atmosphere depends on the value of the actual temperature gradient with respect to the adiabatic gradient. The instability occurs when temperature decays faster than the adiabatic gradient or when potential temperature increases with altitude.

Moist unsaturated thermodynamics.
Virtual temperature.
Boundary layer.

Thermodynamic diagram of water



The terrestrial conditions are such that water is present under its three forms.

Moist air thermodynamics

two gas phases, dry air(d), water vapour (v), one liquid phase (l) and one ice phase (i)

pressure $p = p_d + e$ (water vapour pressure denoted as e)

mass mixing ratio $r = \frac{\rho_v}{\rho_d}, r_l = \frac{\rho_l}{\rho_d}, r_i = \frac{\rho_i}{\rho_d}, r_T = r + r_l + r_i$

$M_d = 29\, g$ $\quad M_v = 18\, g$

$R_d = 287\, J\, kg^{-1} K^{-1}$ $\quad R_v = 461.5\, J\, kg^{-1} K^{-1}$

$C_{pd} = 1005.7\, J\, kg^{-1} K^{-1}$ $\quad C_{pv} = 1870\, J\, kg^{-1} K^{-1}$

$C_l = 4190\, J\, kg^{-1} K^{-1}$ à $T > 0°C$ $\quad C_i = 2106$ à $T \approx 0°C$

$\frac{R_d}{R_v} = \epsilon = 0.622$ $\quad \frac{C_{pv}}{C_{pd}} = \beta = 1.86$ $\quad \kappa = \frac{R_d}{C_{pd}} = 0.285$

$r = \frac{e/(R_v T)}{p_d/(R_d T)} = \epsilon \frac{e}{p-e}$ $\quad p_d = p \frac{\epsilon}{\epsilon + r}$ $\quad e = p \frac{r}{\epsilon + r} = p_d \frac{r}{\epsilon}$

saturation pressure e^s, saturating mixing ratio r^s (function of p and T)

relative humidity $H \equiv \frac{e}{e^s} = \frac{r}{r^s}\left(\frac{1 + r^s/\epsilon}{1 + r/\epsilon}\right)$

specific volume $\alpha \equiv \frac{1}{\rho} = \frac{V_a + V_l + V_i}{m_d + m_v + m_l + m_i} = \alpha_d \left(\frac{1 + r_l(\alpha_l/\alpha_d) + r_i(\alpha_i/\alpha_d)}{1 + r_T}\right) \simeq \frac{\alpha_d}{1 + r_T}$

$$\alpha \simeq \frac{R_d T}{p_d} \frac{1}{1+r_T} = \frac{R_d T}{p} \frac{1+r/\epsilon}{1+r_T}$$

Non saturated air: notion of virtual temperature $\boldsymbol{T_v \equiv T \frac{1+r/\epsilon}{1+r} \simeq T(1+0{,}608\,r)}$

Saturated air: notion of density temperature $\boldsymbol{T_\rho \equiv T \frac{1+r^S/\epsilon}{1+r_T} = T_v \frac{1+r^S}{1+r_T}}$

T_v is the temperature of dry air with the same density as moist air in the unsaturated case : $p=\rho R_d T_v$

T_ρ is the temperature of the dry air with the same density as moist air in the saturated case : $p=\rho R_d T_\rho$

In the tropical regions where r may reach 0,04, the difference between T and T_v may reach 2,5%.

T_v is always larger than T. This is not always true for T_ρ which may be smaller than T when the load in liquid water is high.

In the unsaturated case, $\frac{1}{p}\frac{dp}{dz} = -\frac{g}{R_d T_v}$

Entropy for the moist unsaturated case

We consider here transformations where r is conserved.

Entropies s_d, s_v for the dry part of the air and water vapour:

$s_d = C_{pd} \ln(T/T_0) - R_d \ln(p_d/p_0)$, $s_v = C_{pv} \ln(T/T_0) - R_v \ln(e/p_0)$

Entropy per unit mass of dry air :

$$s = s_d + r\,s_v = (C_{pd} + r\,C_{pv}) \ln(T/T_0) - R_d (1 + r/\epsilon) \ln(p/p_0) + A$$

where we have put in A (calculate it!) a number of constant terms (depending of r).

Defining $s \equiv (C_{pd} + r\,C_{pv}) \ln(\theta/T_0) + A$, the potential temperature is

$$\theta \equiv T \left(\frac{p_0}{p}\right)^{\kappa \frac{1+r/\epsilon}{(1+r\beta)}} \simeq T \left(\frac{p_0}{p}\right)^{\kappa(1-0{,}24r)}.$$

It is conserved for reversible adiabatic unsaturated transformations.

Since r is conserved, T can be replaced by T_v in the above expression.

The virtual potential temperature is defined as

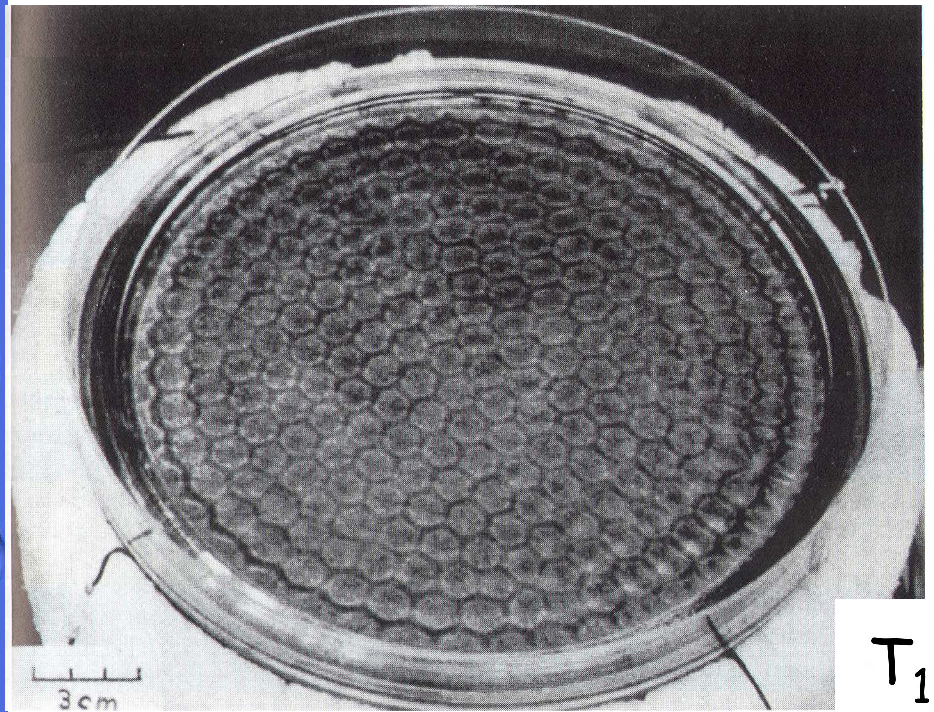
$$\theta_v \equiv T_v \left(\frac{p_0}{p}\right)^{\kappa}.$$

θ_v is very nearly conserved in the same conditions as θ.

Comparing θ_v for two parcel is the same, when they are brought to the same pressue as comparing their virtual temperature and hence their density.

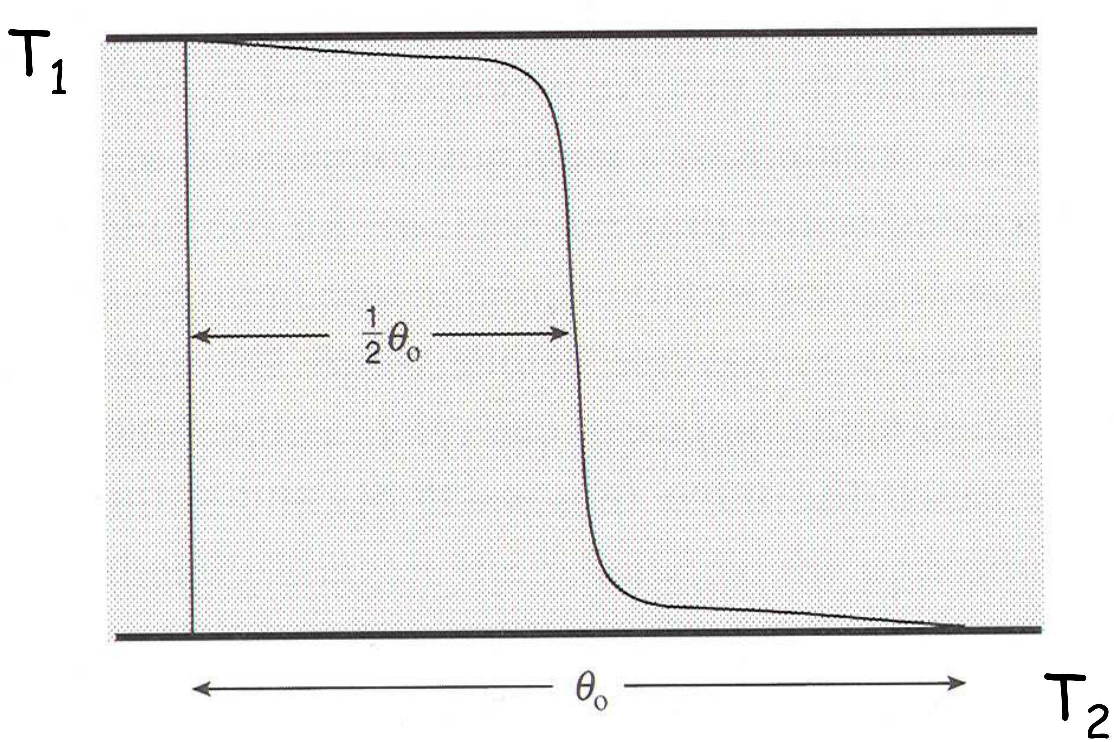
The θ_v profile determines stability for moist unsaturated atmosphere.

Rayleigh-Bénard Convection



Convection between two plates maintainded at temperatures T_1 et T_2. Due to stirring within the cell, a mean quasi-isothermal profile is produced in the interior and the temperature gradient concentrates into two thin boundary layers near the plates.

$T_2 - T_1 = \theta_0$



Hexagonal cells for non turbulent convection.

Temperature profile for turbulent convection.

In the well-stirred zone, temperature is homogenized by generating small scale structures which are eventually smoothed out by diffusion (that means molecular exchanges).

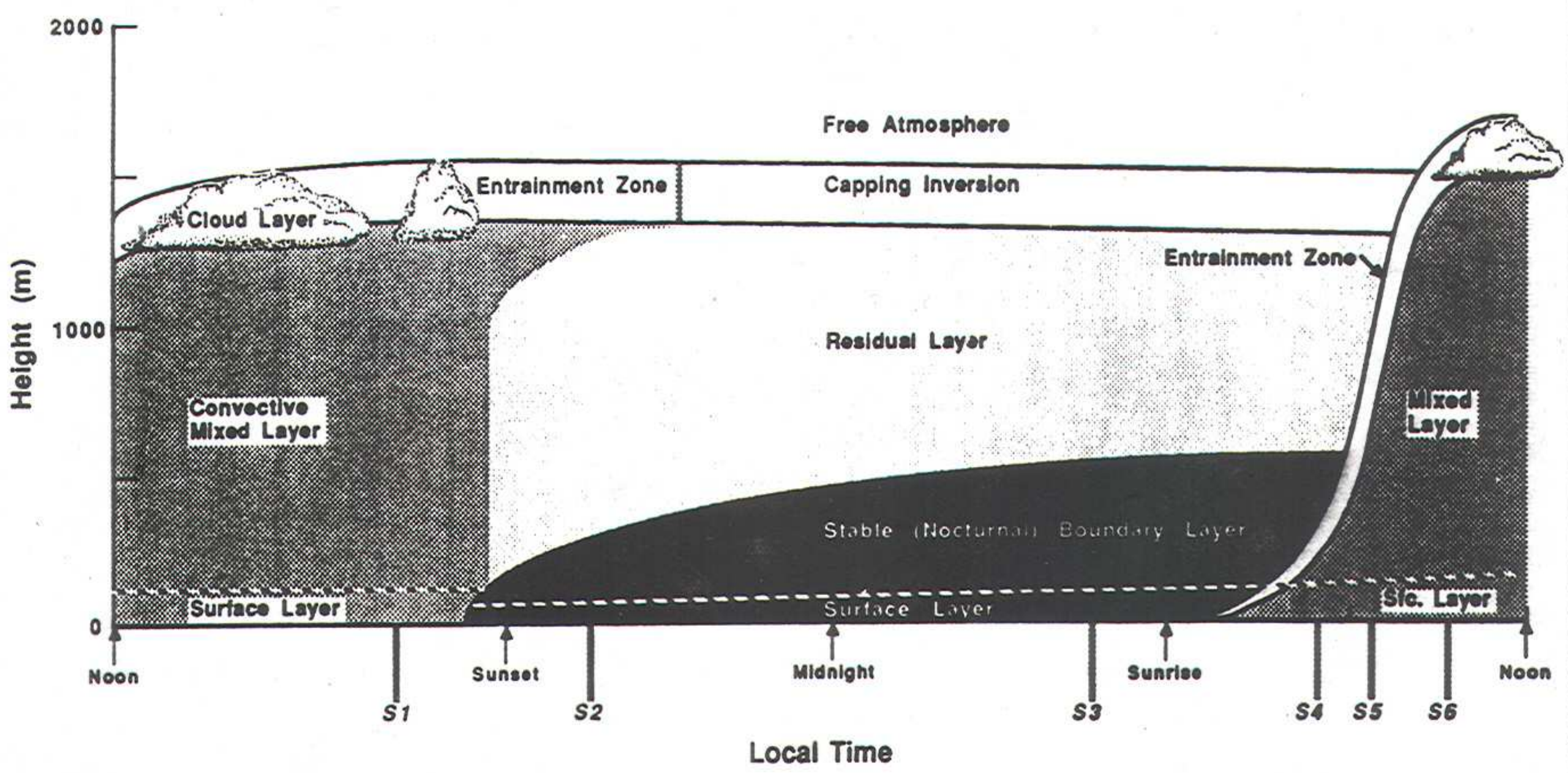
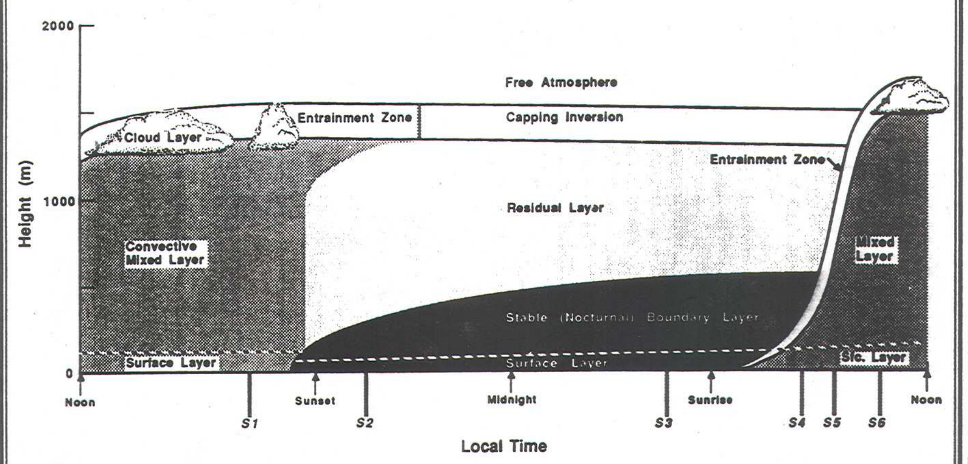


Fig. 1.7 The boundary layer in high pressure regions over land consists of three major parts: a very turbulent mixed layer; a less-turbulent residual layer containing former mixed-layer air; and a nocturnal stable boundary layer of sporadic turbulence. The mixed layer can be subdivided into a cloud layer and a subcloud layer. Time markers indicated by S1-S6 will be used in Fig. 1.12.

Diurnal cycle of a continental boundary layer (mixed layer during the day, stable layer and neutral residual layer during the night)



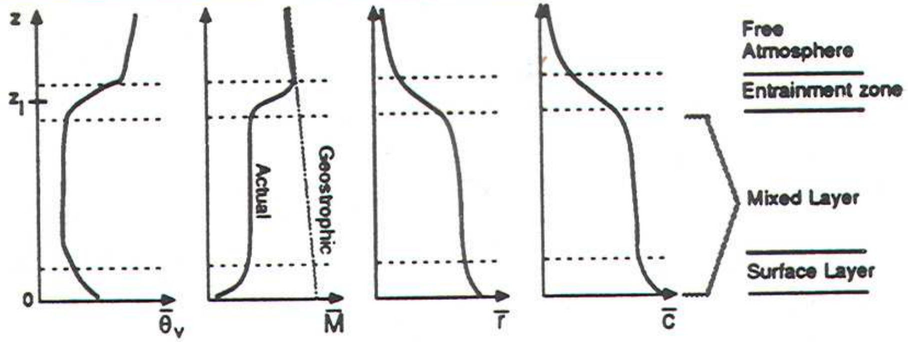
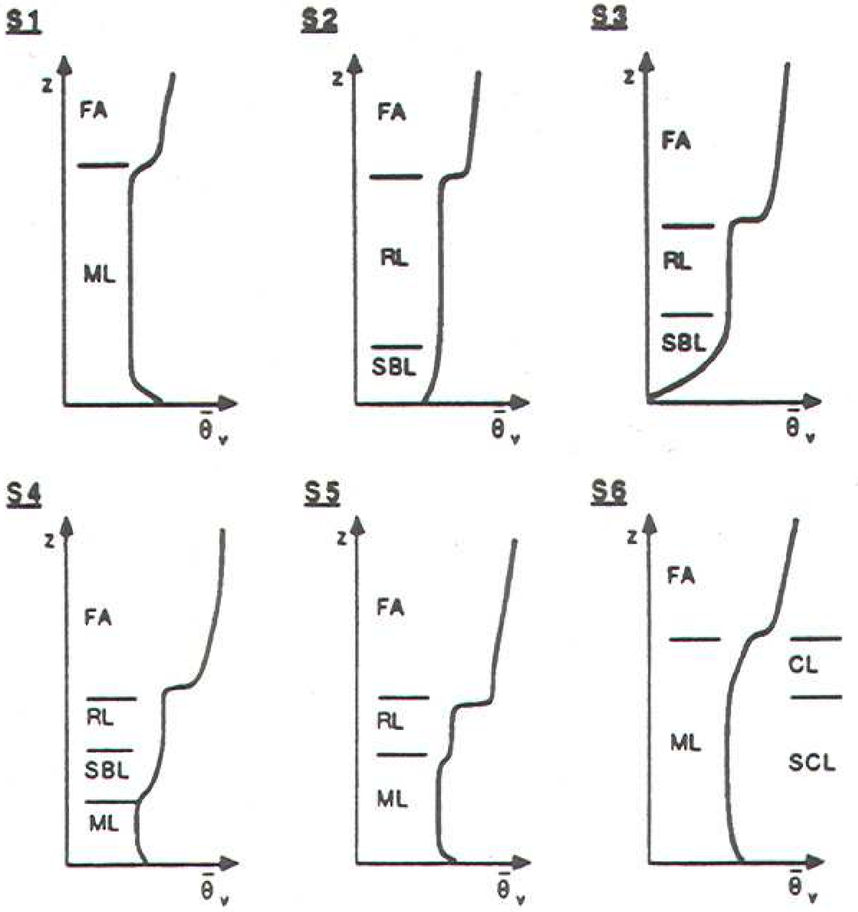


Fig. 1.9 Typical daytime profiles of mean virtual potential temperature $\overline{\theta}_v$, wind speed $\overline{M}$ (where $\overline{M}^2 = \overline{u}^2 + \overline{v}^2$), water vapor mixing ratio $\overline{r}$, and pollutant concentration $\overline{c}$.

Fig. 1.12
Profiles of mean virtual potential temperature, $\overline{\theta}_v$, showing the boundary-layer evolution during a diurnal cycle starting at about 1600 local time. S1-S6 identify each sounding with an associated launch time indicated in Fig. 1.7.



Evolution of temperature profiles during the daily cycle of a continental boundary layer

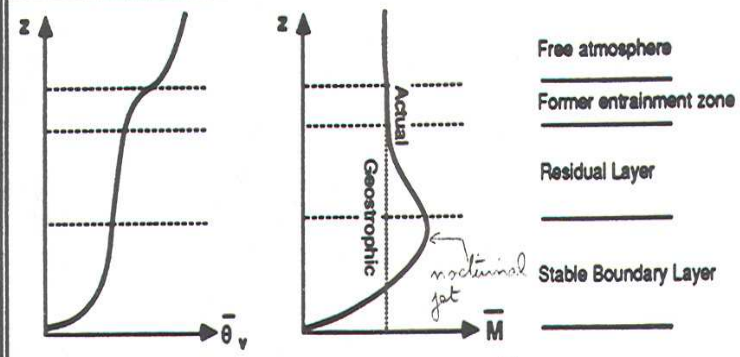
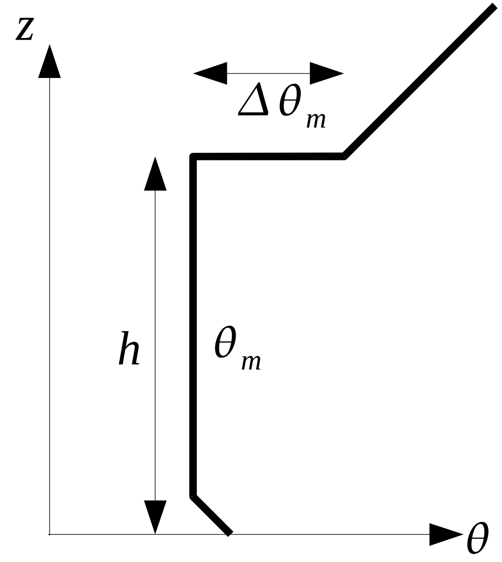


Fig. 1.11 Mean virtual potential temperature, $\overline{\theta}_v$, and wind speed, $\overline{M}$, profiles for an idealized stable boundary layer in a high-pressure region.

Development of an inversion above a boundary layer



Evolution of θ_m by the difference between top and bottom fluxes:

$$h\frac{d\theta_m}{dt}=\overline{w'\theta'}_0-\overline{w'\theta'}_h$$

Growth of h by entrainment:

$$\frac{dh}{dt}=w_e$$

Evolution of inversion gap by entrainment and variation of θ_m:

$$\frac{d\Delta\theta_m}{dt}=w_e\gamma-\frac{d\theta_m}{dt}$$

Equilibration between flux and entrainment at the top of the boundary layer:

$$\Delta\theta_m w_e=-\overline{w'\theta'}_h$$

The solution requires a closure assumption on w_c or $\overline{w'\theta'}_h$

We choose $\overline{w'\theta'}_h=-c_E\phi$ with $\phi\equiv\overline{w'\theta'}_0$

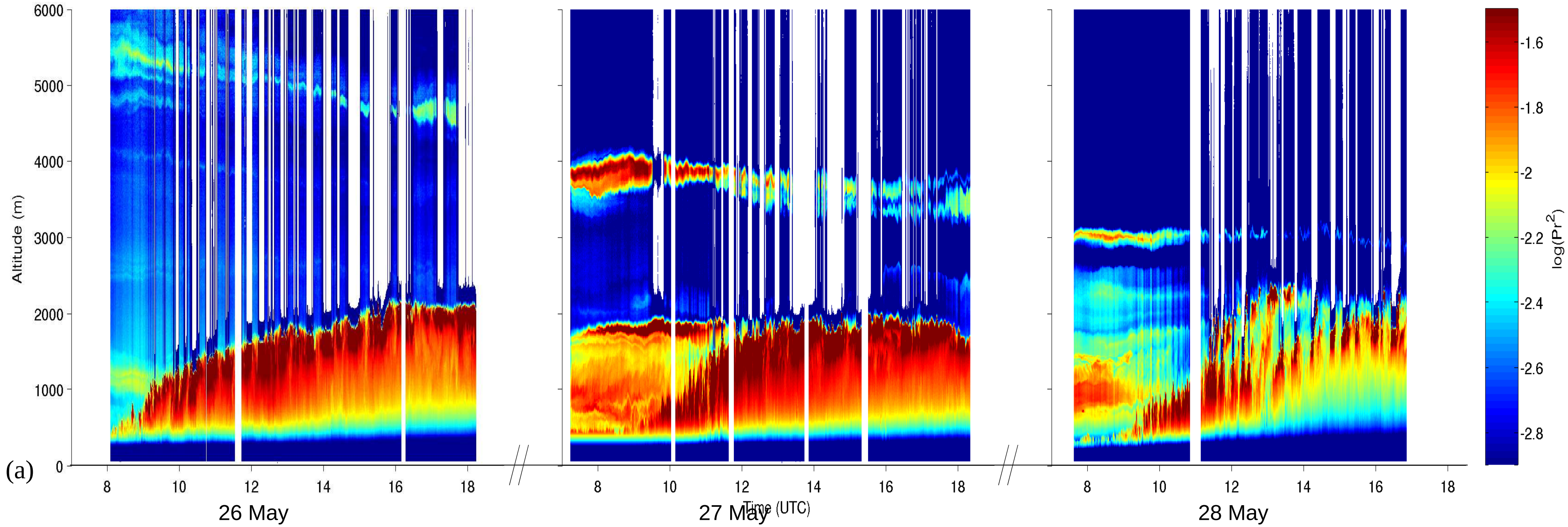
Hence $\frac{d\Delta\theta_m}{dt}=\frac{\gamma c_E\phi}{\Delta\theta_m}-\frac{\phi(1+c_E)}{h}$ and $\frac{dh}{dt}=\frac{c_E\phi}{\Delta\theta_m}$

A solution with $h=\Delta\theta_m=0$ at $t=0$ is obtained as $\Delta\theta_m=At^{1/2}$ and $h=Bt^{1/2}$

where $A=\sqrt{\frac{2\gamma c_E^2\phi}{1+2c_E}}$ and $B=\sqrt{\frac{2\phi(1+2c_E)}{\gamma}}$. We get also: $\theta_m=\theta_0+\sqrt{\frac{2(1+c_E)\gamma\phi t}{1+2c_E}}$

The inversion layer which caps convective motion is a direct consequence of the development of a boundary layer

Observation of the boundary layer aerosols + volcanic ashes by lidar
(SIRTA)



TO RETAIN

- The atmosphere is heated by its bottom. A boundary layer is generated above the ground over a depth of 1000 à 4000m (following season and latitude). In this boundary layer, convective mixing maintains a zero gradient of virtual potential temperature except in a thin surface layer where motion is inhibited.
-
- Under continental conditions, the boundary layer undergoes a strong diurnal cycle with an active mixing layer during day time followed by a restratification from the surface during the night where a residual neutrally stratified layers persists aloft.

Moist air thermodynamics and the generation of clouds

Moisture condensation

The saturation partial pressure depends on the temperature through the Clausius-Clapeyron law $d \ln(e_S)/dT = L/R_v T^2$

Approximate formula (in hPa)

$e_s^{liquide} = 6{,}112 \exp(17{,}67\ T/(T+243.55))$

$e_s^{glace} = \exp(23{,}33086 - 6111{,}72784/T + 0{,}15215 \ln(T))$

Exemples of saturating ratios

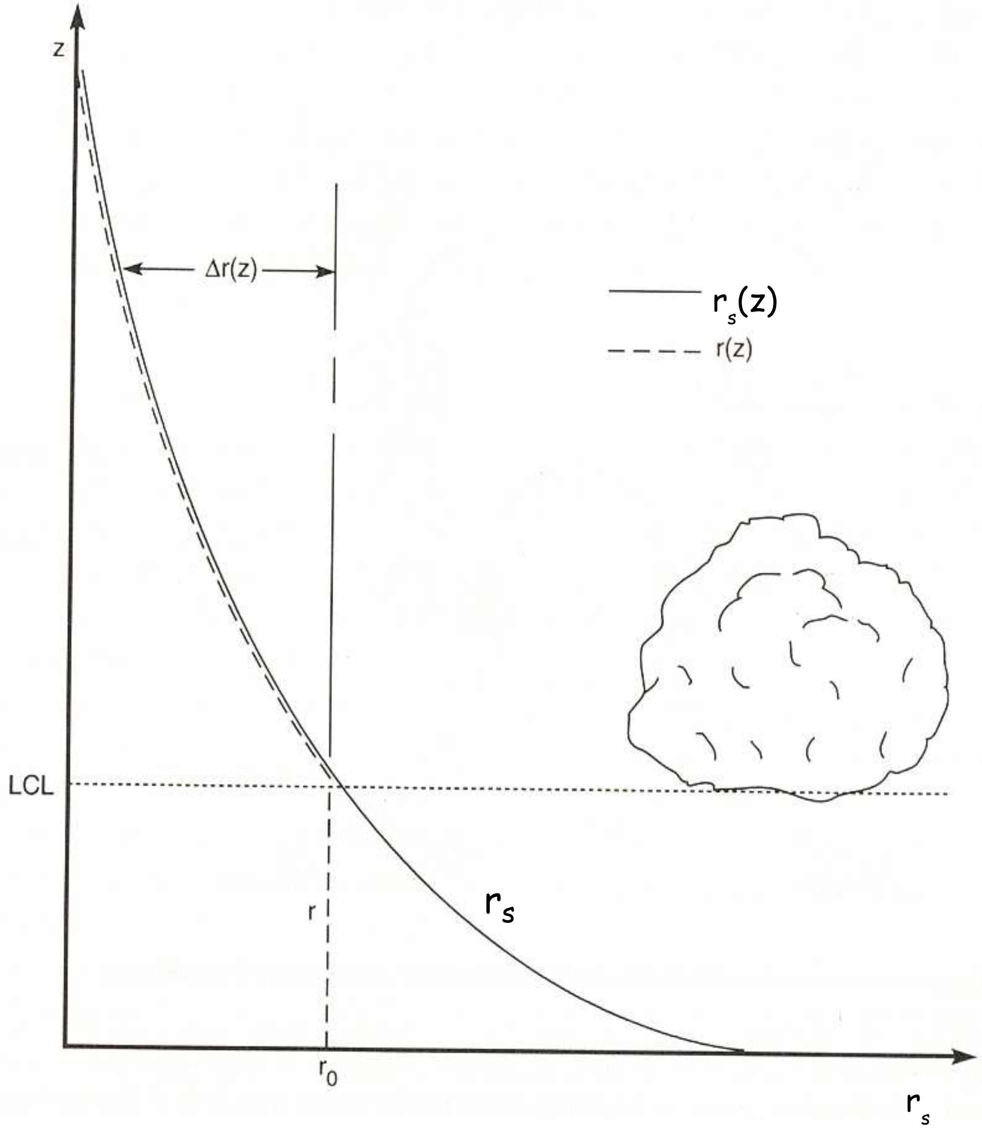
at 1000hPa and T=20°C: r_s = 14,5 g/kg,

at 800 hPa (2000m) and T = 7°C: r_s =7,8 g/kg,

at 500 hPa and T=-30°C r_s =0,47 g/kg,

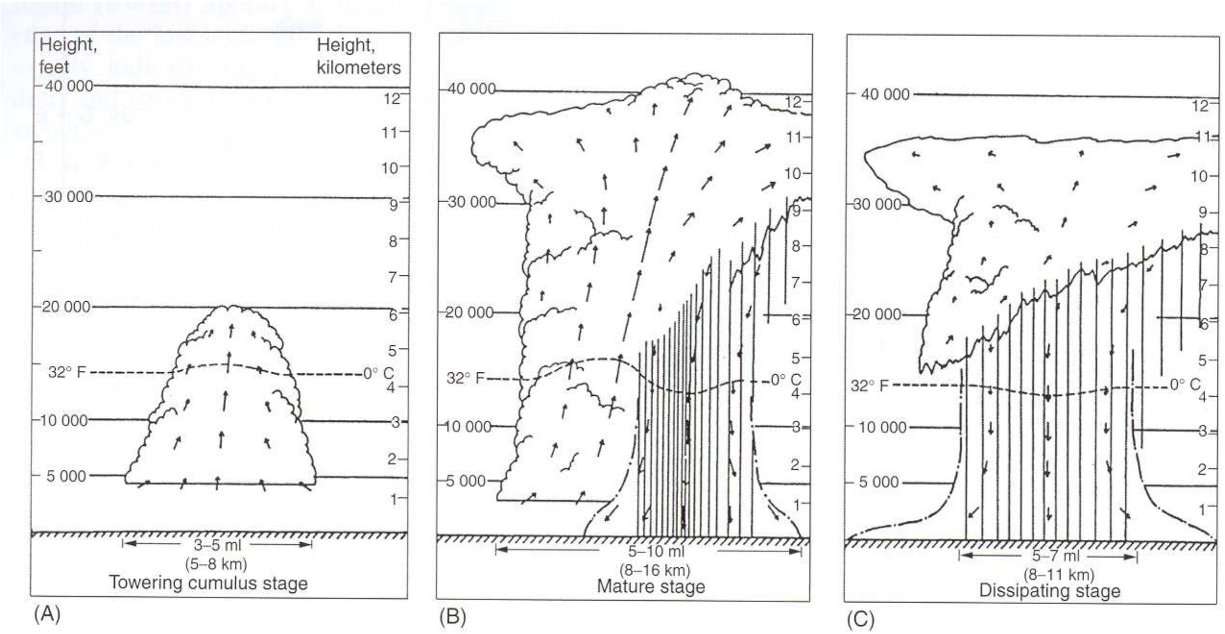
at 100 hPa and T =-80°C r_s =0,003 g/kg,

(the atmospheric water content is divided by approximately 4 orders of magnitude between the ground and 100 hPa in the tropics)

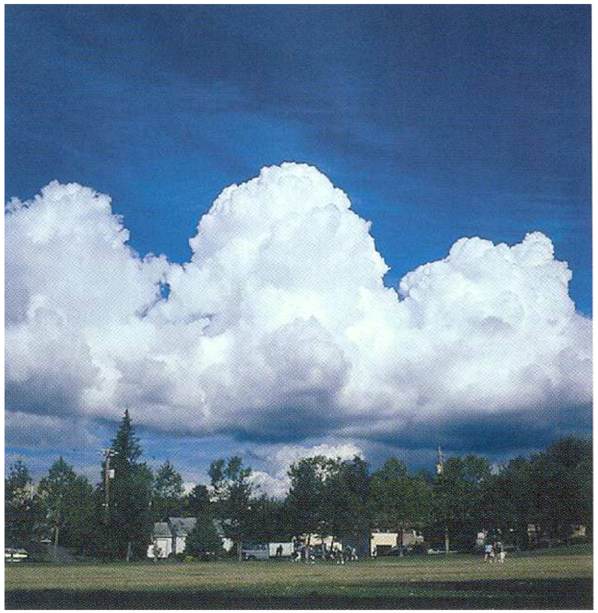


LCL (lifting condensation level): level at which parcels rising from the ground condensate

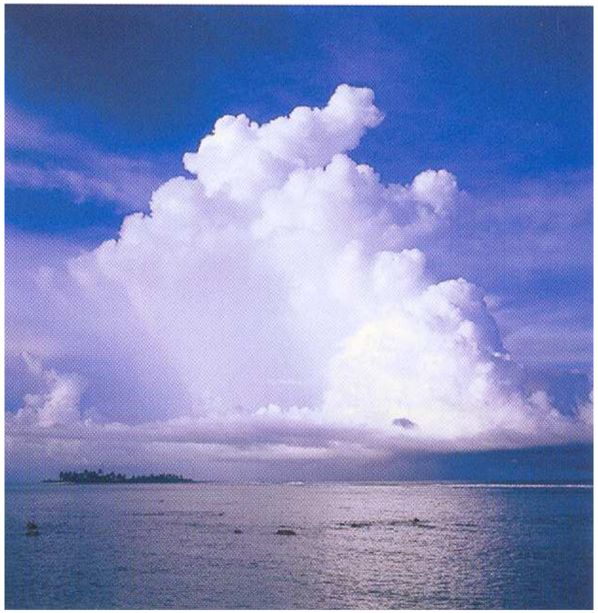
Formation of convective clouds

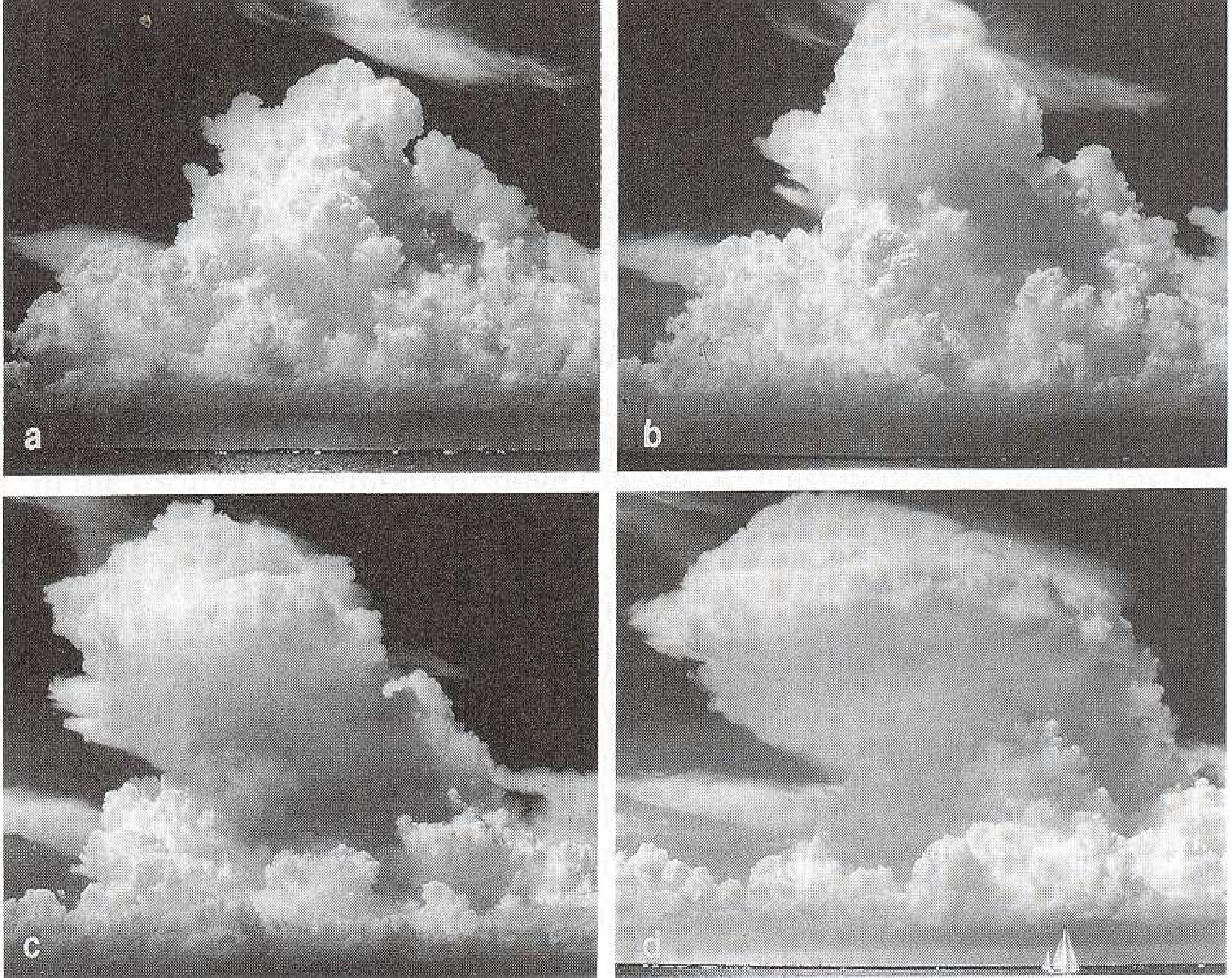


cumulus



cumulonimbus




a
b
c
d

Lifting condensation level for a parcel lifted from the surface

Let define the relative humidity as $H=e/e_{S}$. Saturation is reached when $H=1$.

We have $d\ln(H)=d\ln(e)-d\ln(e_S)$

Using the conservation of potential temperature $d\ln(e)=d\ln(p)=\frac{1}{\kappa}\frac{1+r\beta}{1+r/\epsilon}d\ln(T)$

Using Clapeyron and Kirkhhoff laws: $d\ln(e_S)=\frac{L}{R_v T^2}dT=\frac{L_0+(C_{pv}-C_l)(T-T_0)}{R_v T}d\ln(T)$

Hence $-\ln(H)=\left(\frac{1}{\kappa}\frac{1+r\beta}{1+r/\epsilon}+\frac{C_l-C_{pv}}{R_v}\right)\ln\frac{T^*}{T}+\left(\frac{L_0+(C_l-C_{pd})T_0}{R_v}\right)\left(\frac{1}{T^*}-\frac{1}{T}\right)$

where T^* is the temperature at the LCL.

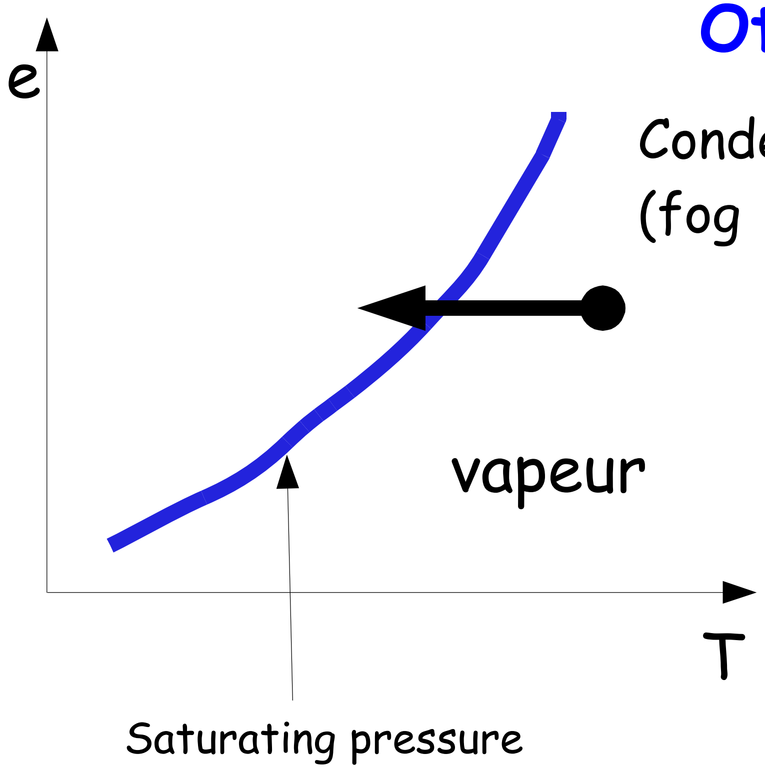
Approximate solution $T^*=\frac{2840}{3.5\ln(T)-\ln(e)-4.805}+55$ with e in hPa.

The pressure at the LCL p^* is then given by $\ln\frac{p^*}{p}=\frac{1}{\kappa}\frac{1+r\beta}{1+r/\epsilon}\ln\frac{T^*}{T}$

and the altitude at the LCL can be determined by integrating $\frac{dT}{dz}=\frac{-g}{C_{pd}}\frac{1+r}{1+r\beta}$

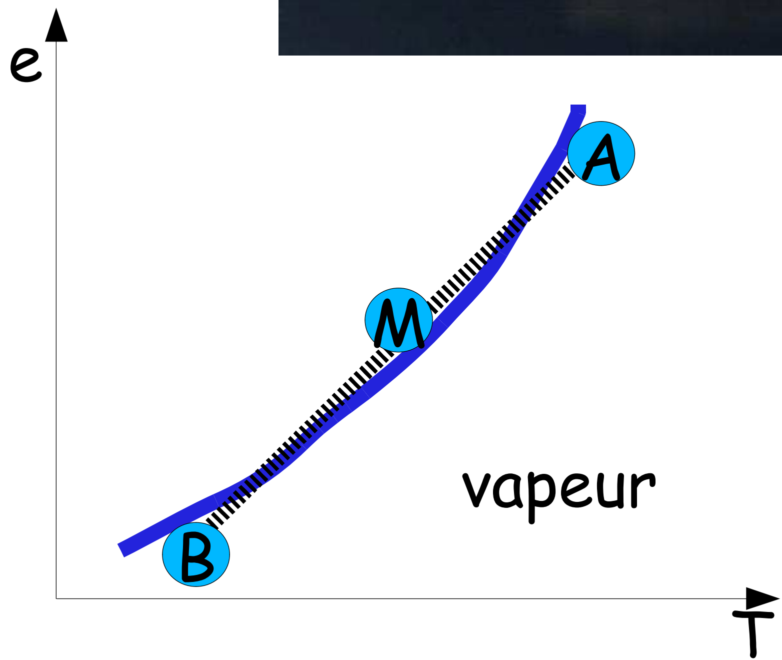
hence $z^*-z=\frac{C_{pd}}{g}\frac{1+r\beta}{1+r}(T-T^*)$

Other forms of condensation



Condensation by isobaric cooling
(fog and dew point)

Condensation by mixing of warm moist air (A) with cold dry air (B) (generation of contrails and fog above lakes)



Other types of clouds

Altitude clouds

Cirrus

Composed of ice, rarely opaque.
Are formed above 6000m in mid-latitudes.
They are often precursors of a warm front.
In the tropics are formed as remains of anvils or by in situ condensation of rising air, up to the tropopause.

Alto-cumulus

Contain liquid droplets between 2000 and 6000 m in mid-latitudes. Cluster into compact herds. They are often, during summer, precursors of late afternoon and evening developments of deep convection.

Other types of clouds
Low stratiform clouds

Strato-cumulus

Composed by water droplets, opaque or very opaque, base under 2000m, associated with weak precipitations

Nimbo-stratus

Very opaque low clouds, undefined base, associated with persistent precipitations, snow by cold weather

Stratus

Low clouds with small opacity, undefined base under 2000m or at the ground (fog)

TO RETAIN

- Atmospheric moisture is limited by the Clausius-Clapeyron relation which fixes the saturation mixing ratio as a function of temperature and pressure.

- When an air parcel is rising, it reaches the lifting condensation level where its water content is saturating. Condensation occurs defining the basis of cumuliform clouds.

- Other mechanisms may lead to generation of clouds like isobaric cooling (generation of morning fog) or the mixing between warm humid air and cold dry air (aircraft contrails, fog over lakes during winter).

Equivalent potential temperature and the potential instability

Moist air thermodynamics (cont'd): liquid + vapour

Equilibrium of the temperature T, the pressure $p=e^S$, and the free energy g among the two phases, with $g=u+p\alpha-T\,s=h-T\,s$, $h=u+p\alpha$, u and h being only function de T for an ideal gast. Latent heat $L=h_v^S-h_l=T(s_v^S-s_l)$

Loi de Kirchhoff

$$d\,L=dT\left[\left(\frac{\partial h_v}{\partial T}\right)_p-\left(\frac{\partial h_l}{\partial T}\right)_p\right]+de^S\left[\left(\frac{\partial h_v}{\partial p}\right)_T-\left(\frac{\partial h_l}{\partial p}\right)_T\right]$$

Ideal gas law

$$=dT[C_{pv}-C_l]+d\,e^S\left[-\alpha_l-e^S\left(\frac{\partial \alpha_l}{\partial p}\right)_T\right]$$

Negligeable volume of the liquid phase

$$\boldsymbol{\frac{d\,L}{dT}=C_{pv}-C_l}$$

vaporisation $L_0=2{,}5\times10^6$ J kg^{-1} à 0°C.

Loi de Clausius-Clapeyron

For a variation of the equilibrium between the two phases: $d\,g_v=d\,g_l$

Using the definition of g and the first law of thermodynamics $T\,ds=du+p\,d\,\alpha$

$$-s_v\,dT+\alpha_v\,de^S=-s_l\,dT+\alpha_l\,d\,e^S$$

$$\frac{d\,e^S}{dT}=\frac{s_v-s_l}{\alpha_v-\alpha_l}=\frac{L}{T(\alpha_v-\alpha_l)}\approx\frac{L\,e_S}{R_v T^2}$$

Equivalente potential temperature

For a parcel of humid air, the entropy per unit mass of dry air is est

$s = s_d + r\, s_v + r_l\, s_l$

- # $s_d = C_{pd} \ln(T/T_0) - R_d \ln(p_d/p_0)$ for the dry air,
- # $s_v = C_{pv} \ln(T/T_0) - R_v \ln(e/p_0)$ for water vapour,
- # $s_l = C_l \ln(T/T_0)$ for liquid water.

Using $L = T(s_v^S - s_l)$ and $H = e/e^S$,and after a few manipulations:

$$s = s_d + r\, s_v + r_l\, s_l = s_d + r(s_v - s_v^S) + r(s_v^S - s_l) + r_T\, s_l = s_d + r_T\, s_l + \frac{L\, r}{T} + r(s_v - s_v^S)$$

$$= (C_{pd} + r_T C_l) \ln(T/T_0) - R_d \ln(p_d/p_0) + \frac{L\, r}{T} - r\, R_v \ln(H)$$

The equivalent potential temperature θ_e can be defined such that

$$d\, s = (C_{pd} + r\, C_l)\, d \log(\theta_e/T_0)$$

hence

$$\boldsymbol{\theta_e = T \left(\frac{p_0}{p_d}\right)^{R_d/(C_{pd}+r_T C_l)} (H)^{-r\, R_v/(C_{pd}+r_T C_l)} \exp\left(\frac{L\, r}{(C_{pd}+r_T C_l)\, T}\right)}$$

This quantity is conserved under both saturated and non saturated moist adiabatic transforms where condensates are carried aloft.

For a saturated parcel, $\theta_e = T \left(\frac{p_0}{p_d}\right)^{R_d/(C_{pd}+r_T C_l)} \exp\left(\frac{L\, r^S}{(C_{pd}+r_T C_l)\, T}\right)$

function of (T, p_d, r_T).

Instability under the presence of moisture

The conserved quantity for a moist saturated adiabatic is the equivalent potential temperature

$$\theta_e(T,p) \approx \theta \exp \frac{L r^S(T,P)}{C_p T}$$

when r_T is small

$$\frac{\partial \theta_e}{\partial T} = \frac{\theta_e}{T}\left(1 - \frac{L r^S}{C_p T}\right) > 0$$

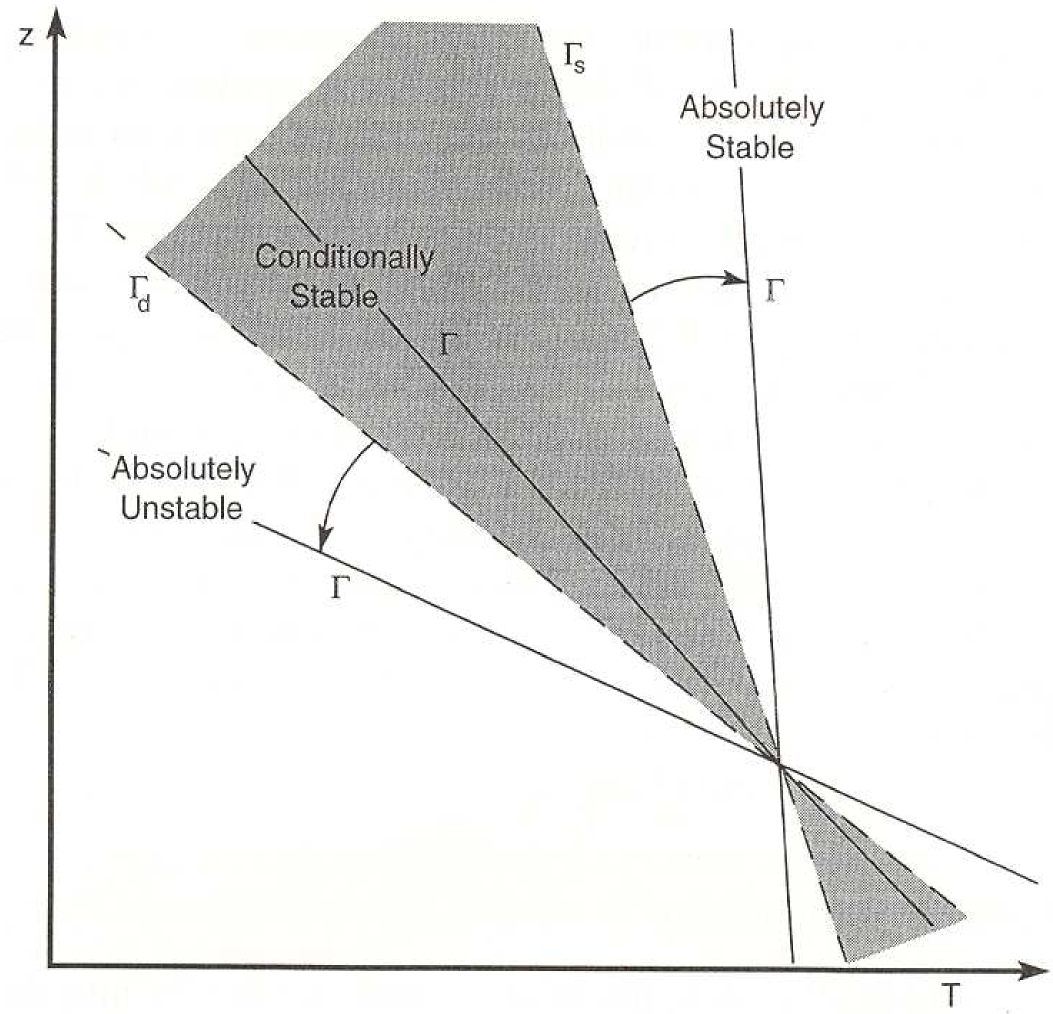
Instability conditions compared to that of dry air .

The instability for unsaturated air where $d\,\theta_e/dz > 0$ is potential because it does not show up until the air is saturated.

Simplification: we neglect the effect of water vapour on air density (virtual temperature effect)

Γ_d : dry adiabatic

Γ_S: saturated moist adiabatic



Complementary note: Simplified calculation of the saturated moist gradient

In a saturated adiabatic transform, and for a unit mass of dry air:

$$(C_p + r^s C_{pv} + r_l C_{pl})\, dT + L\, d\, r^s - R_d T\, d \log p_d - r^s R_v\, d \log e^S = 0.$$

(neglected terms in green)

Using the ideal gas law,

$$R_d T\, d \log p_d = \frac{1}{\rho_d} dp = -g\, dz.$$

Then we need to write the variation of r^S as a function of T et p :

$$d\, r^s = \left(\frac{\partial r^s}{\partial T}\right) dT + \left(\frac{\partial r^s}{\partial p}\right) dp.$$

Using once again the hydrostatic law, we obtain :

$$\left(C_p + L\left(\frac{\partial r^s}{\partial T}\right)\right) = -g\left(1 - \rho L\left(\frac{\partial r^s}{\partial p}\right)\right) dz,$$

hence

$$\Gamma_S = \Gamma_d \frac{1 - \rho L\left(\frac{\partial r^s}{\partial p}\right)}{1 + \frac{L}{C_p}\left(\frac{\partial r^s}{\partial T}\right)} \approx \Gamma_d \frac{1 + \frac{L r^S}{R_d T}}{1 + \frac{L^2 r^S}{R_v T^2}}$$

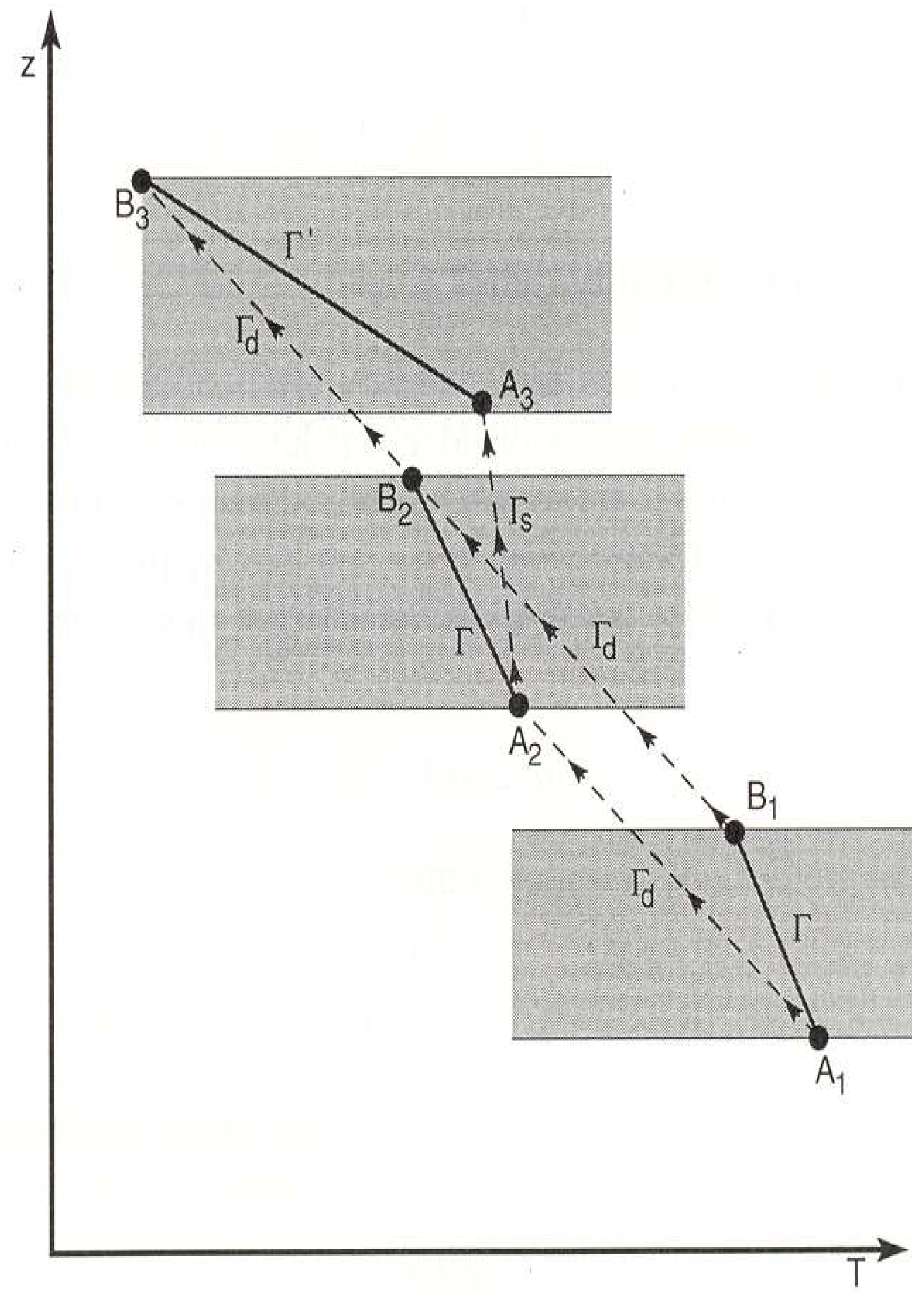
Potential instability

Potential instability appears when

$\frac{d\theta}{dz} > 0$ but $\frac{d\theta_e}{dz} < 0$

It is realised when a potentially unstable layer is lifted,for instance by crossing some orographic zone ou due to frontal transport.

as soon as the first bottom layer gets saturated, convection is initiated.



Pseudo-equivalent potential temperature and the conditional instability

Pseudo-equivalent potential temperature

Within a fast ascent in a convective cloud, liquid water forms droplets which are big enough to fall as precipitaion and are not entrained by the lofted air.
A useful approximation of this process is the pseudo-equivalent transform where the heat capacity of liquid water is neglected and all the heat generated by condensation is absorbed by the gas phase. Hence the variation of the entropy per unit mass of dry air is:

$$ds_p = ds + C_l (r - r_T)\ d\ln(T)$$

which is integrated as:

$$s_p = s - C_l \int_T^{T_0} (r - r_T)\, d\ln(T')$$

or $e^{s_p} \equiv \left(\frac{\theta_{ep}}{\theta_0}\right)^{(C_{pd}+rC_l)} = \left(\frac{T}{T_0}\right)^{(C_{pd}+r_T C_l)} \left(\frac{p_0}{p}\right)^{R_d} e^{\frac{Lr}{T}} H^{-rR_v} e^{-C_l\int_T^{T_0}(r-r_T)\, d\ln(T')}$

$$e^{s_p} = \left(\frac{T}{T_0}\right)^{C_{pd}} \left(\frac{p_0}{p}\right)^{R_d} e^{\frac{Lr}{T}} H^{-rR_v} e^{-C_l\int_T^{T_0} r\, d\ln(T')}$$

$$\boldsymbol{\theta_{ep} = T \left(\frac{p_0}{p_d}\right)^{R_d/C_{pd}} (H)^{-rR_v/C_{pd}} \exp\left(\frac{Lr}{C_{pd}T}\right) \exp\left(-\frac{C_l}{C_{pd}}\int_T^{T_0} r\, d\ln(T')\right)}$$

Notice that for $T > T^*$, $r = r_T$ is preserved and does not depend on T whereas for $T < T^*$, we have $r = r^S(T, p_d)$ and the integral is along a pseudo-adiabatic path.

Pseudo-equivalent potential temperature (cont'd)

$$\theta_{ep}=T\left(\frac{p_0}{p_d}\right)^{R_d/C_{pd}} H^{-r\,R_v/C_{pd}} \exp\left(\frac{L\,r}{C_{pd}\,T}\right)\exp\left(-\frac{C_l}{C_{pd}}\int_{T^*}^{T} r\,d\ln(T')\right)$$

θ_{ep} is conserved for adiabatic transform up to the LCL followed by a pseudo-adiabatic.

θ_{ep} is function de T, p_d, r for unsaturated air and is only function of T and p_d for saturated air.

In practice, except in very moist tropical regions, θ_{ep} differs weakly from θ_e and can be used also in reversible transforms for which liquid water is transported with the parcel.

Practical approximative formula θ_{ep} (Bolton, 1980)

$$\theta_{ep}=T\left(\frac{1000}{p}\right)^{0,2854(1-0,28r)} \exp\left[r(1+0,81r)\left(\frac{3376}{T^*}-2,54\right)\right]$$

This expression is accurate up to 0.3K within the range of atmospheric conditions.

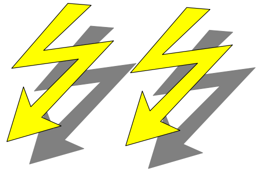
Saturation pseudo-equivalent potential temperature

This temperature is defined for unsaturated ambiant air and is the pseudo-equivalent temperature for saturated air at the same temperature and pressure as the ambiant air

$$\theta_e^* = \theta_{ep}(T, p_d, r^S(T, p_d)) = T\left(\frac{p_0}{p_d}\right)^{R_d/C_{pd}} \exp\left(\frac{L r^S(T, p_d)}{C_{pd} T}\right) \exp\left(\frac{C_l}{C_{pd}} \int_{T^*}^{T} r^S(T, p_d)\, d\ln(T')\right)$$

This temperature is only function of T and p_d

For a saturated ambiant air, it is identical to θ_{ep}



This temperature determines the onset condition of deep convection. The comparison between unsaturated ambiant air and a rising saturated parceal conserving θ_{ep} cannot be done on θ_{ep} because the moisture contribution to this quantity is different for the ambiant air and the rising parcel. If the ambiant air is brought to the same saturation conditions as the rissing parcel, it is guaranteed that an equality between θ_e^* of the ambiant air and θ_{ep} of the rising parcel leads to an equality of temperatures.

In the same way an inequality between θ_e^* and θ_{ep} leads to an inequality of the same sign between temperatures because $\frac{\partial \theta_e^*}{\partial T} > 0$ (check it!).

We neglect here the effects of a lower density of water vapour with respect to dry air. Such effects are less important than those related to latent heat within a convective cloud. They can be taken into account in a more complete theory (Emanuel's book)

Conditional instability

Typical convective situation in the topical region

When an air parcel is displaced verticaly, it first rises along a dry adiabatic and hence reaches its condensation level (LCL).
It then continue to rise as a saturated parcel following a pseudo-adiabatic path.
Next it meets its neutral buoyancy level (LFC) when its température θ_e équals θ_e^* of the ambiant air.

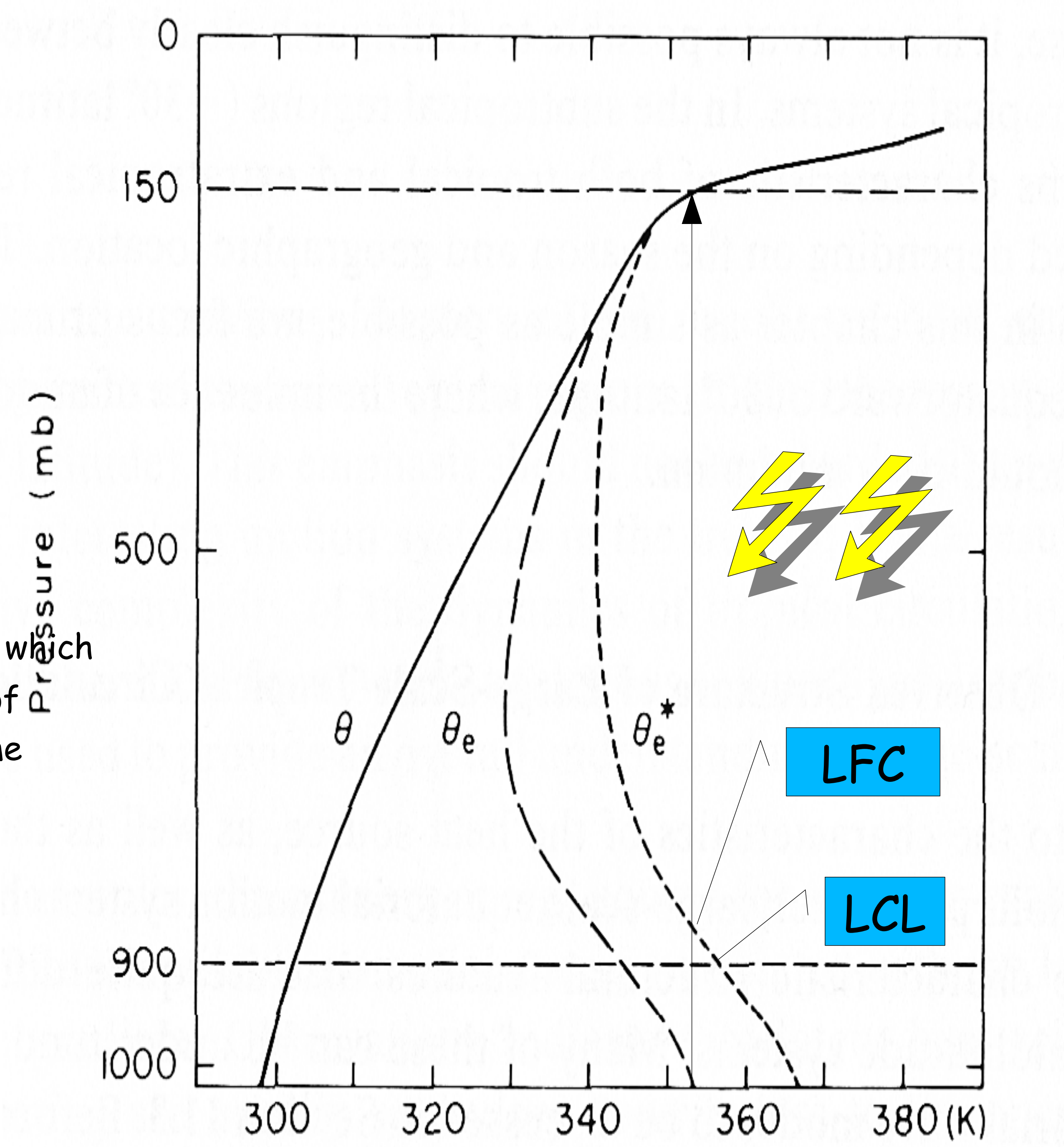
At this time, the parcel temperature equals that of the ambiant air.

The ascent continues if $\frac{d\,\theta_e^*}{d\,z}<0$

Hence it is the profile θ_e^* and non that of θ_e which determines the stability since an inequality of the saturated temperature leads, at the same pressure, to an inequality of the same type on the temperatures since

$$\frac{\partial \theta_e^*}{\partial T}>0$$

Notice: effect of moisture upon density is neglected.



TO RETAIN

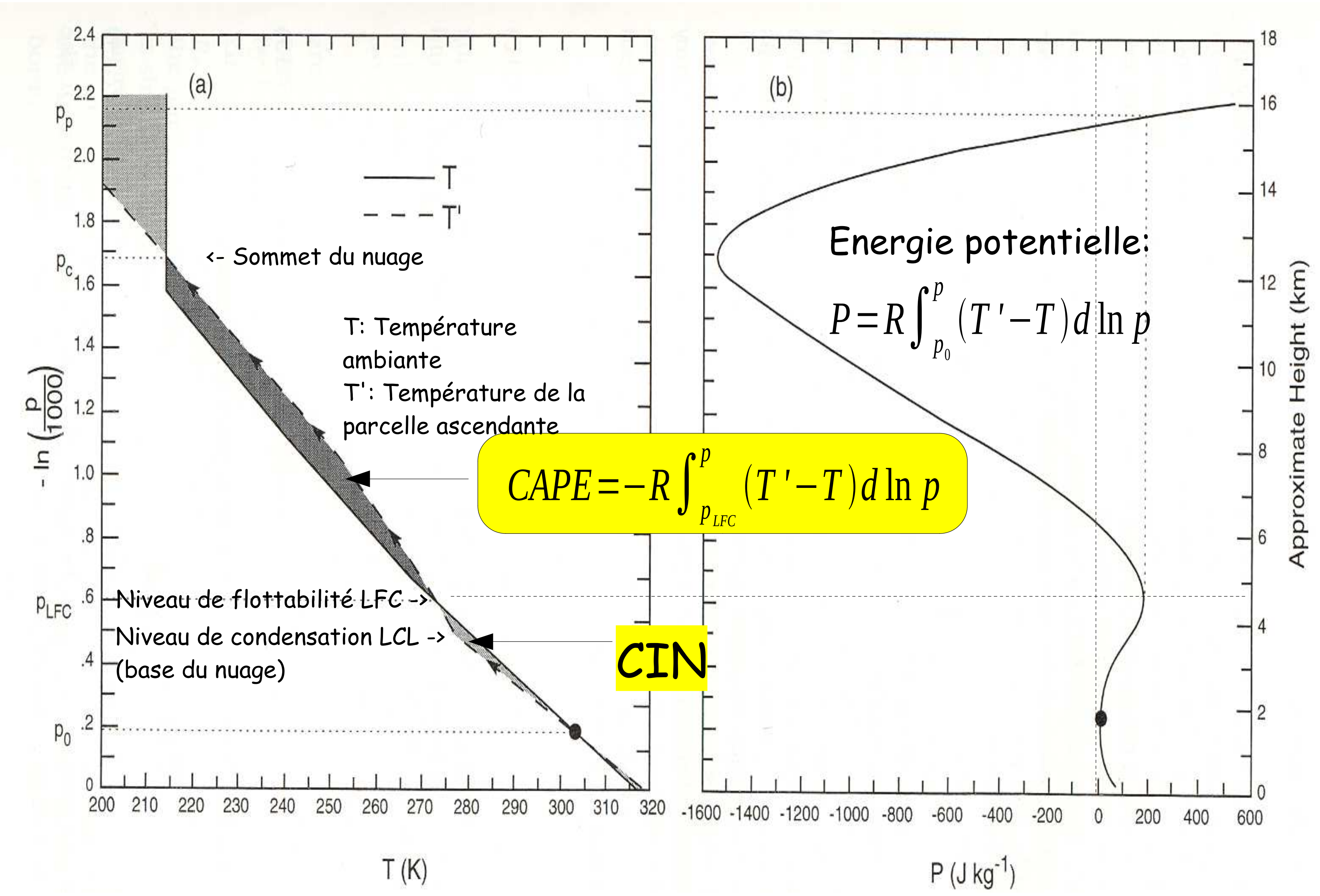
- Inside a cloud, the ascending motion of a non mixing parcel is described by an adiabatic transform (if condensates are lifted with the parcel) or a pseudo-adiabatic transform if condensates precipitate.
-
- Thermodynamic variables can be built for these transforms which generalize the potential temperature defined for dry air.
-
- For saturated air, the instability conditions of dry air are easily generalized using the equivalent potential temperature
-
- In unsaturated air, the instability determined by the profile of pseudo-equivalent potential temperature is conditional. The condition is that an initial finite perturbation brings parcels located near the ground to their level of condensation and then to their level of neutral buoyancy.

Conditional versus potential instability

Conditional instability is associated with the motion of a parcel which needs to reach its neutral buoyancy level to become buoyant.
Potential instability when a whole layer of fluid becomes unstable to moist convection as it rises under the effect of a constrain (orography, sea breeze, ...)

CIN, CAPE
and meteorological diagram

History of a parcel during its ascent within a cloud

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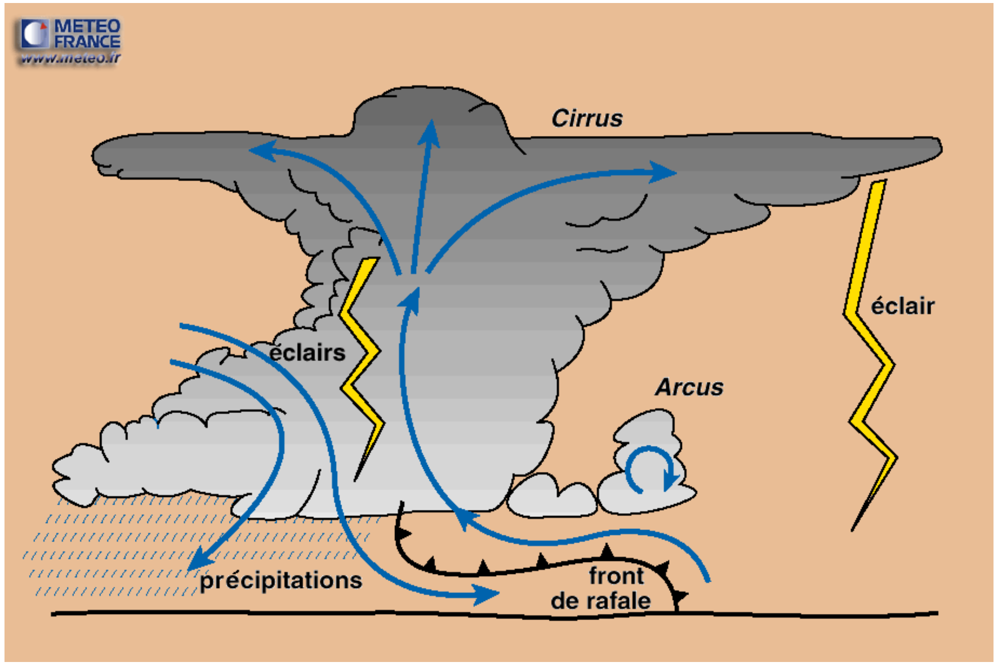
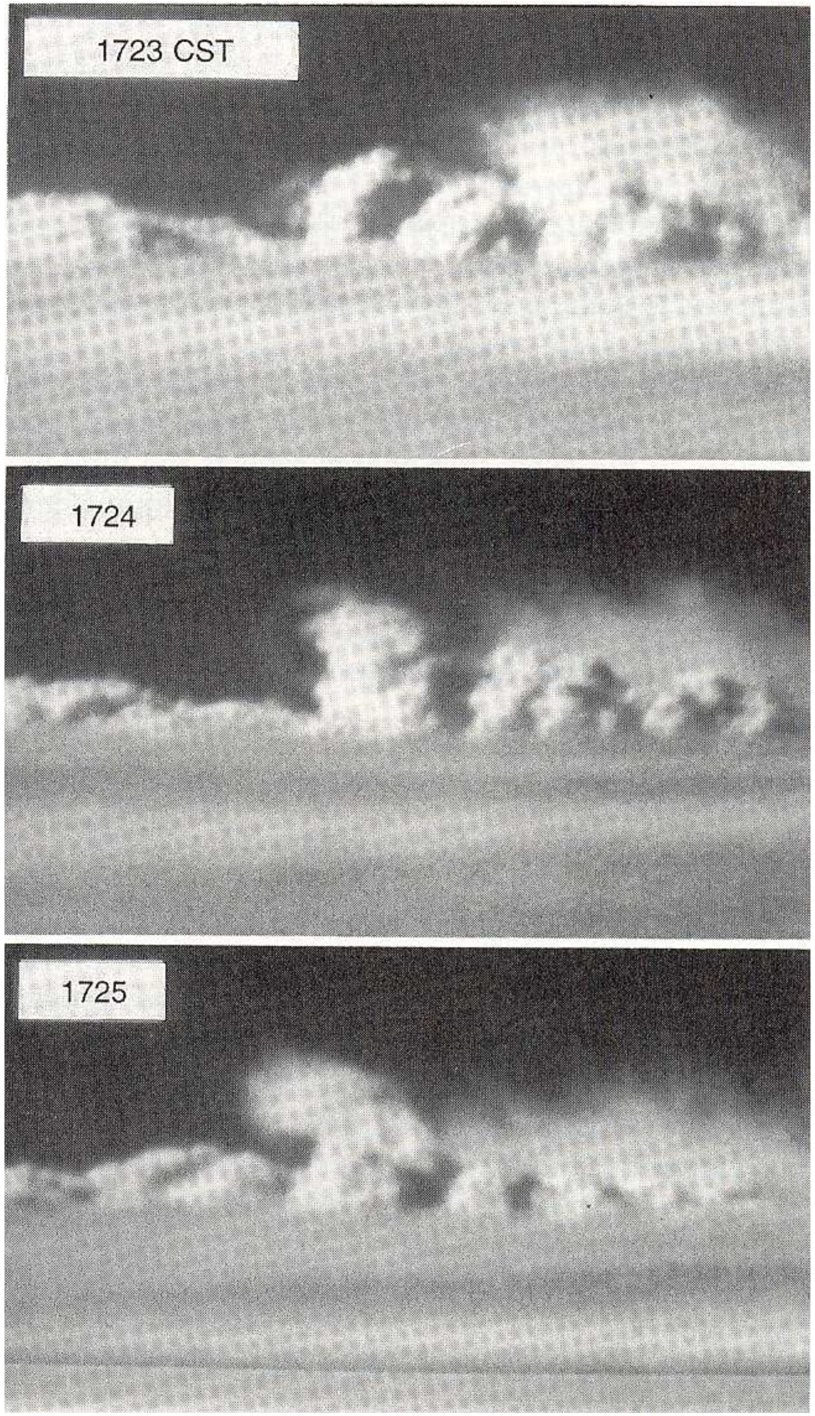
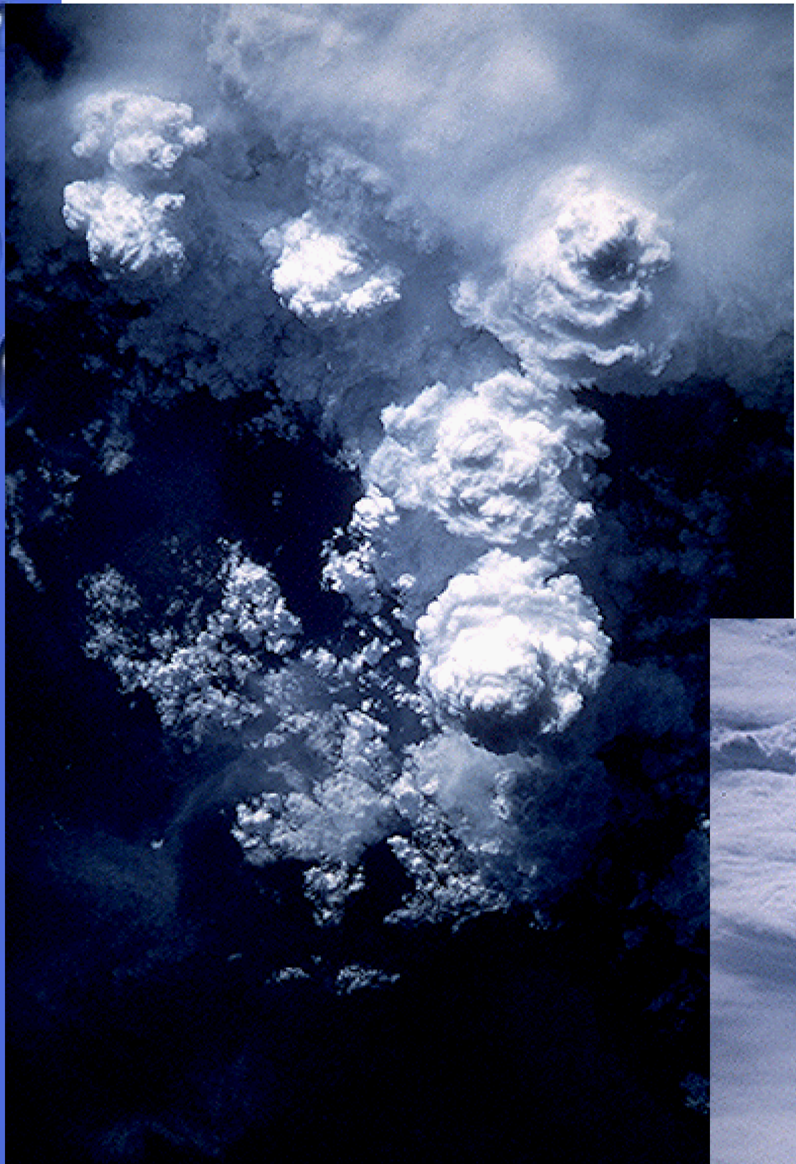
Convective available potential energy (CAPE)

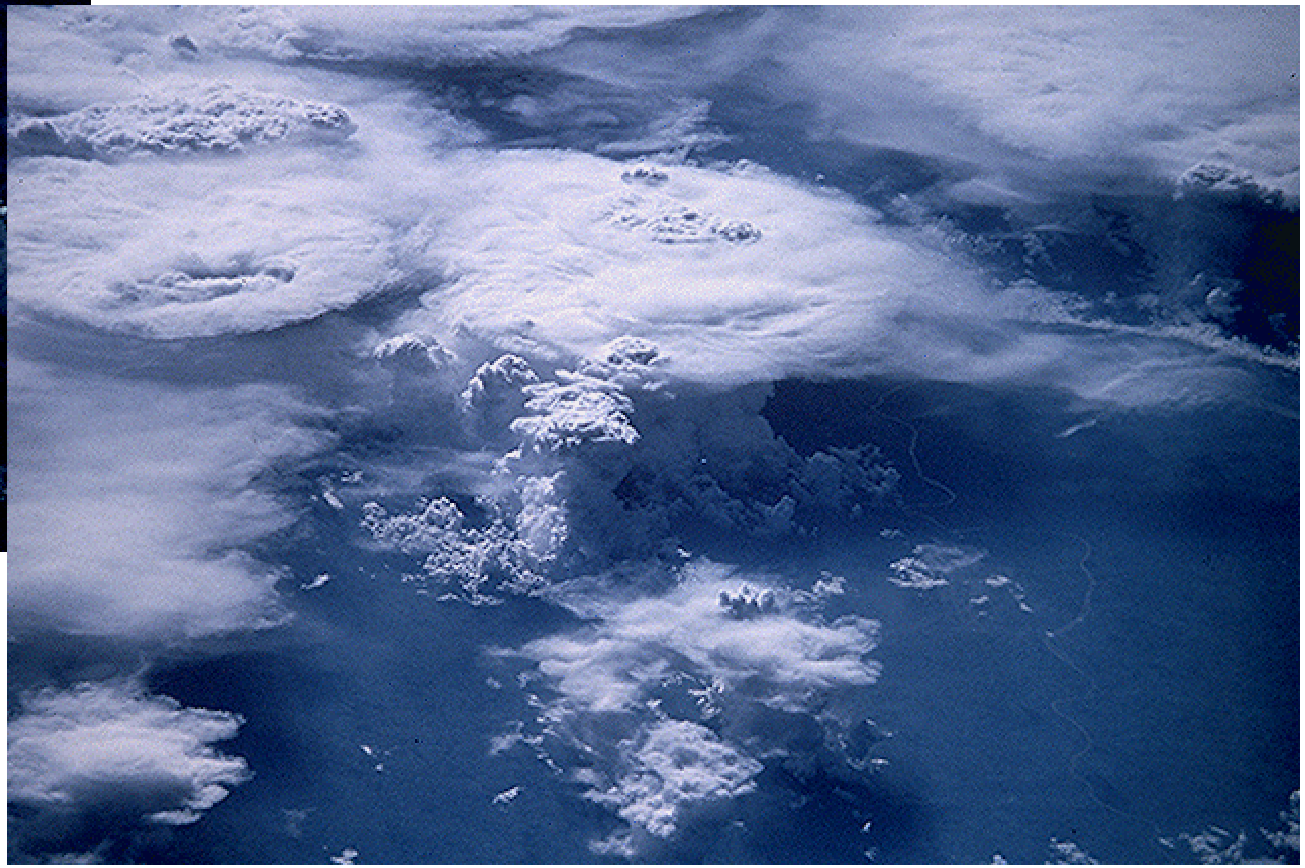
$$CAPE = P(p_{LFC}) - P(p_c)$$

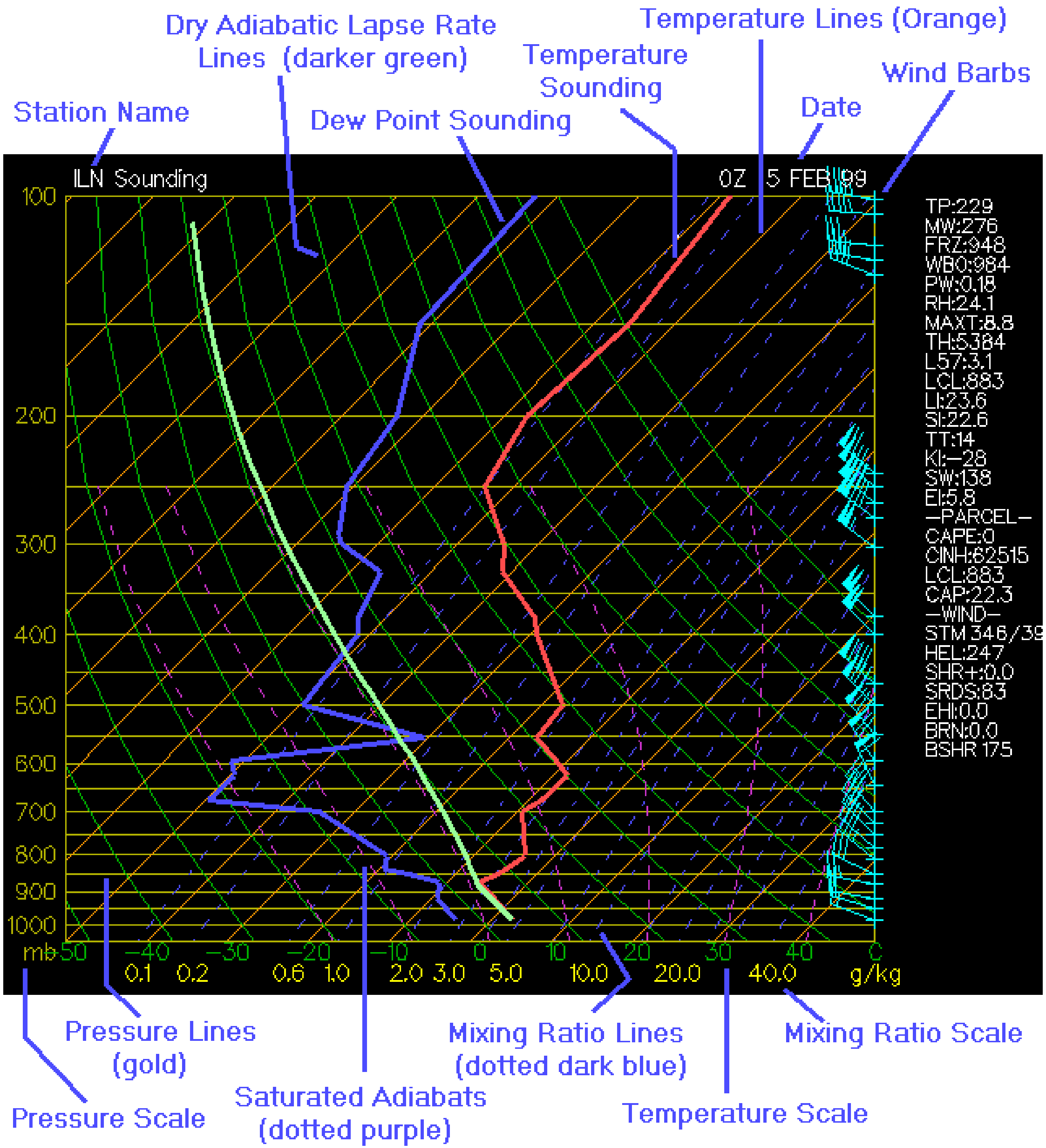




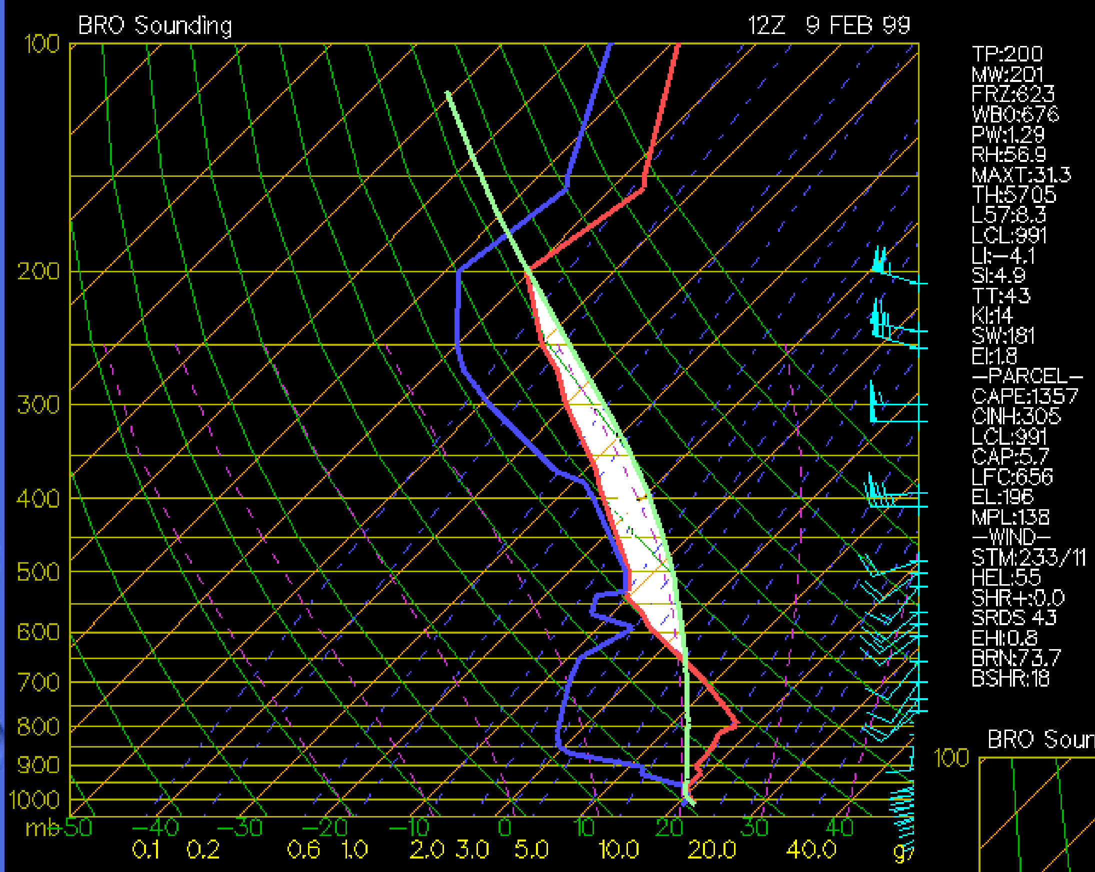


Convectives clouds above Brazil
(pictures taken from a space shuttle)



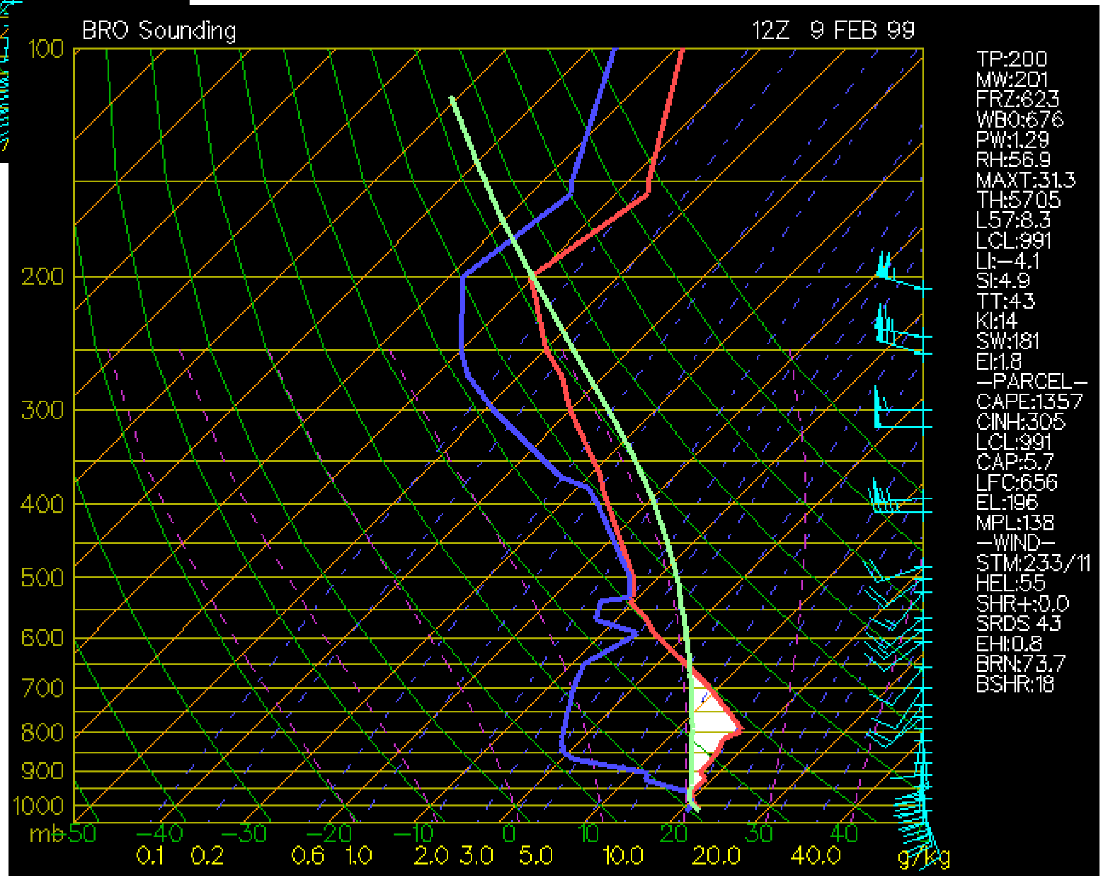


Meteorological diagram representing a sounding.


BRO Sounding
12Z 9 FEB 99
TP:200
MW:201
FRZ:623
WBO:676
PW:1.29
RH:56.9
MAXT:31.3
TH:5705
L57:8.3
LCL:991
LI:-4.1
SI:4.9
TT:43
KI:14
SW:181
EI:1.8
-PARCEL-
CAPE:1357
CINH:305
LCL:991
CAP:5.7
LFC:656
EL:196
MPL:138
-WIND-
STM:233/11
HEL:55
SHR+:0.0
SRDS 43
EHI:0.8
BRN:73.7
BSHR:18

CAPE

CIN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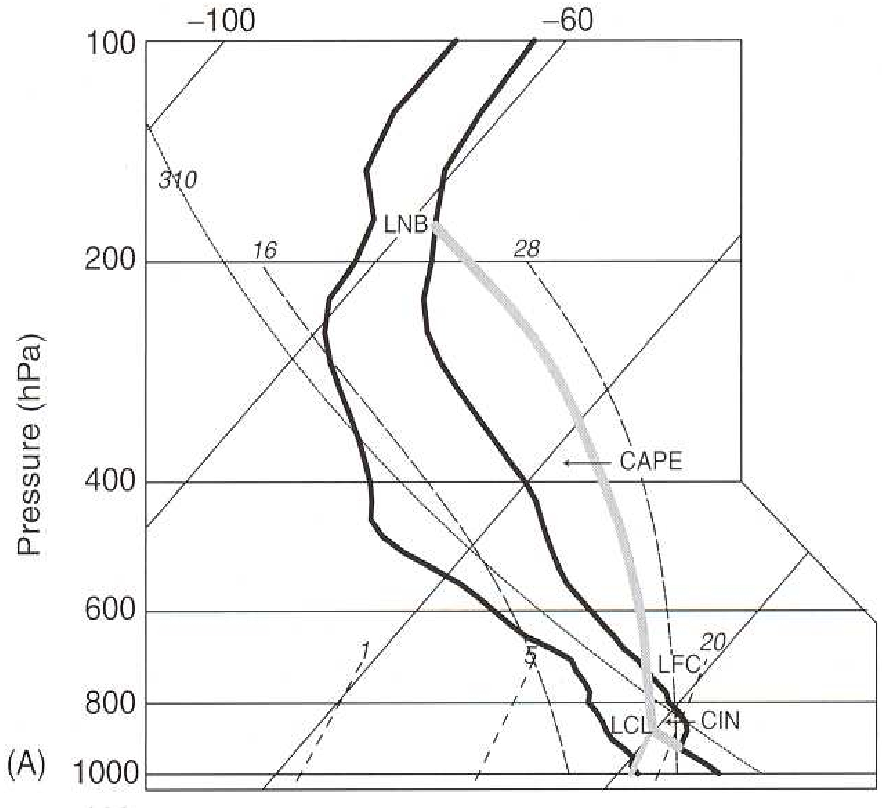
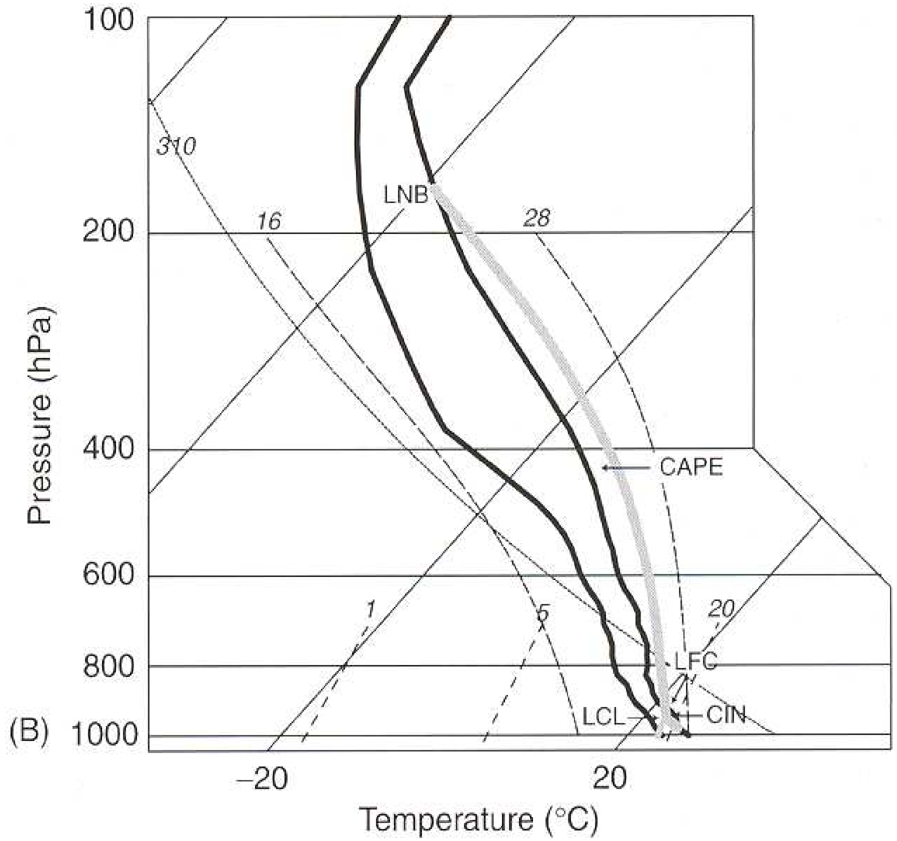
CAPE and meteoroloical diagram



Mid-latitudes : summer situation in continental zone
High CAPE (3000-4000 J kg^{-1}) and high CIN inhibiting the onset of convection.

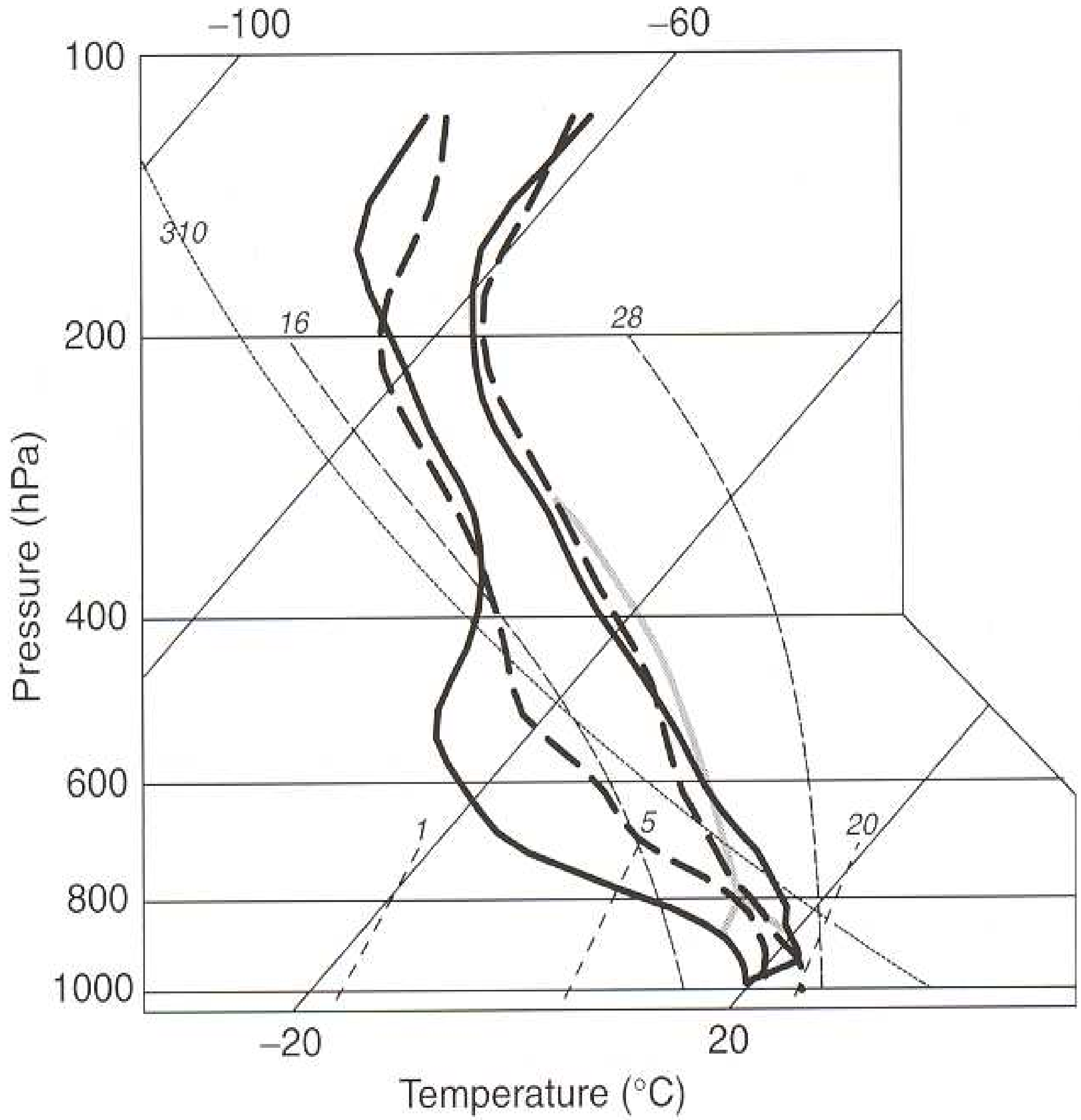
Tropical latitude: oceanic situation
Moderated CAPE (1000-2000 J kg^{-1}) ad weak CIN favoring the onset



Exemple of situation evolving towards potential instability

Solid: beginning
Dash: end

Saturation and destabilisation of the layer 800-870 hPa



TO RETAIN

• Meteorological diagram displays the thermodynamic state of an atmospheric column and allows to estimate its instability potential as well as the levels of cloud basis and cloud top.

•

• The CAPE is the energy which can be released during the ascent motion of a parcel from its free buoyant level to the top of the cloud. It measures the intensity of deep convection. The scending currents may reach values of several tens of m/s.

• The CAPE is weaker for maritime tropical convection than for continental convection in tropics and extra tropics but the onset of convection is easier in the first case due to smaller CIN.

To retain (cont'd)

- We have neglected that a cloud is a mixture of air entrained and detrained over its whole depth. However, the parcels reaching the highest altitude are generally coming from the subcloud region without being diluted.

- The cumulonimbus generated by deep convection are not the only type of clouds. Low stratiform clouds and high altitude cirrus are a good part of cloud cover and paly an important role in the Earth radiative budget. However, deep convection is responsible of the strongest precipitations and hence of most of atmospheric heating by latent heat transfer.

An example of large-scale cloud parameterization

We start from the definition of total enthalpy:

$$k = k_d + r\,k_v + r_l\,k_l + r_i\,k_i = k_d + r\,(k_v - k_l) + (r + r_l + r_i)\,k_l + r_i\,(k_i - k_l)$$

$$k = k_d + r\,L_v + r_T\,k_l - r_i\,L_f = (C_{pd} + r_T\,C_l)\,T + r\,L_v - r_i\,L_f$$

By applying the Kirchhoff law, we can substract the constant $L_v - (C_{pv} - C_l)\,T$ to get:

$$k_{il} = k - L_v - (C_{pv} - C_l)\,T = (C_{pd} + r_T\,C_l)\,T + (-r_l - r_i)\,L_v - r_i\,L_f$$

$$k_{il} = C_{pd} + r_T\,C_l ¿ T - r_l\,L_v - r_i\,L_s$$

Both k and k_{il} are conserved under an adiabatic isobaric process.

Under more general conditions, the heat exchange is (for a unit mass of dry air):

$$\delta Q \equiv d\,h_{il} = d\,k_{il} - \alpha_d\,dp$$

after neglecting the volume of the condensed phase.

The total condensed water static energy h_{il} is conserved for general adiabatic reversible processes.

using $\alpha_d = \alpha(1 + r_T)$, the hydrostatic relation and dividing h_{il} by $1 + r_T$,

we obtain the total condensed virtual static energy: $h_{vil} = \dfrac{C_{pd} + r_T\,C_{pv}}{1 + r_T} - \dfrac{r_l\,L_v + r_i\,L_s}{1 + r_T} + g\,z$

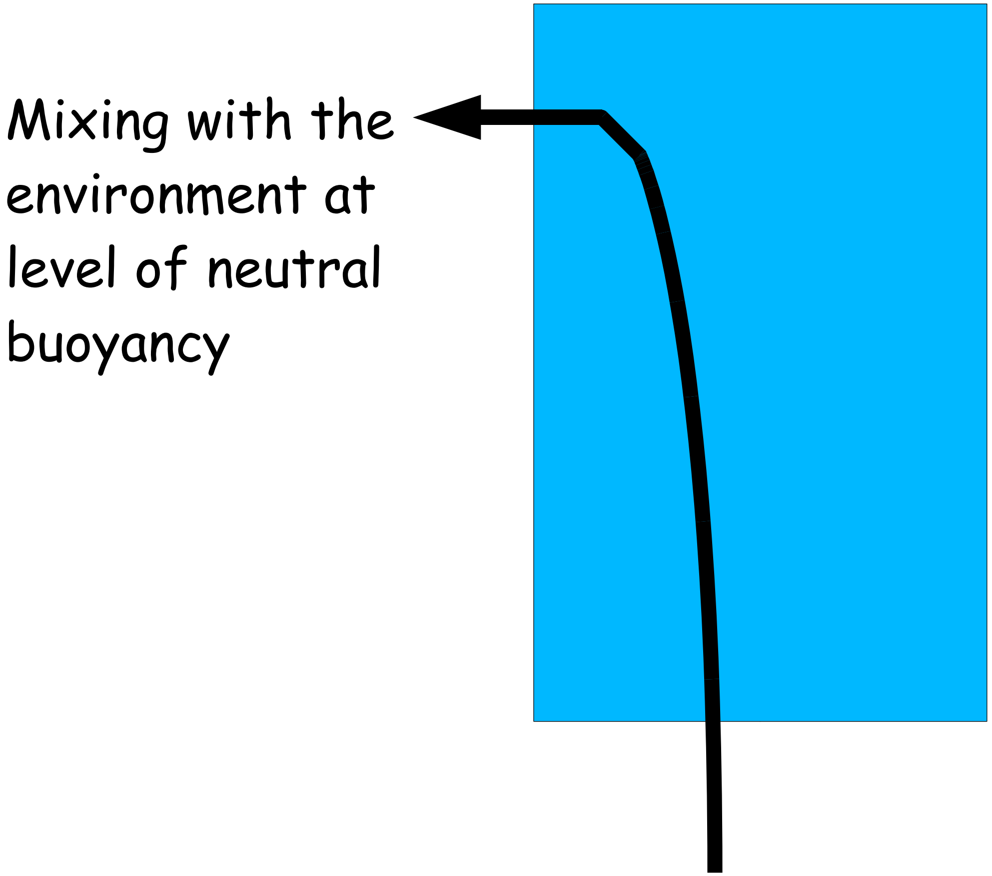
which, in the absence of condensates, reduces to

$$h_v = \frac{C_{pd}\,(1 + \beta\,r)}{1 + r} \approx C_{pd}\,T_v + gz$$

The comparison of two parcels with no condensates is equivalent to compare their virtual temperature and hence their buoyancy.

h_{vil} is conserved for adiabatic and hydrostatic processes with no loss of mass.

This can exploited to determine the altitude reached by a convective updraft assuming that it detrains at the altitude where it is neutrally buoyant when all the condensates have evaporated.



Convective parameterization for large-scale model
K. Emanuel scheme (Emanuel, JAS, 1991; Emanuel & Zivkovic-Rothman, JAS 1999)

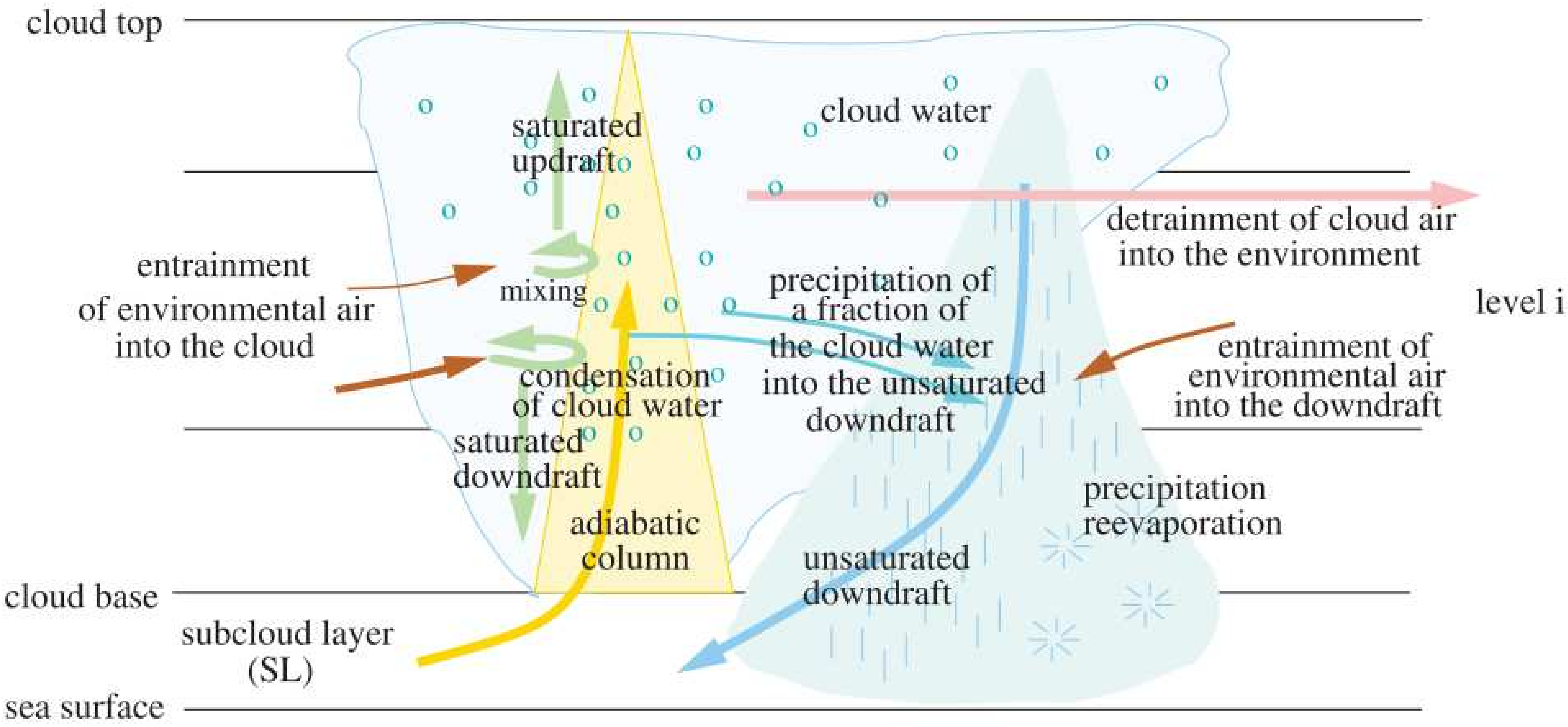
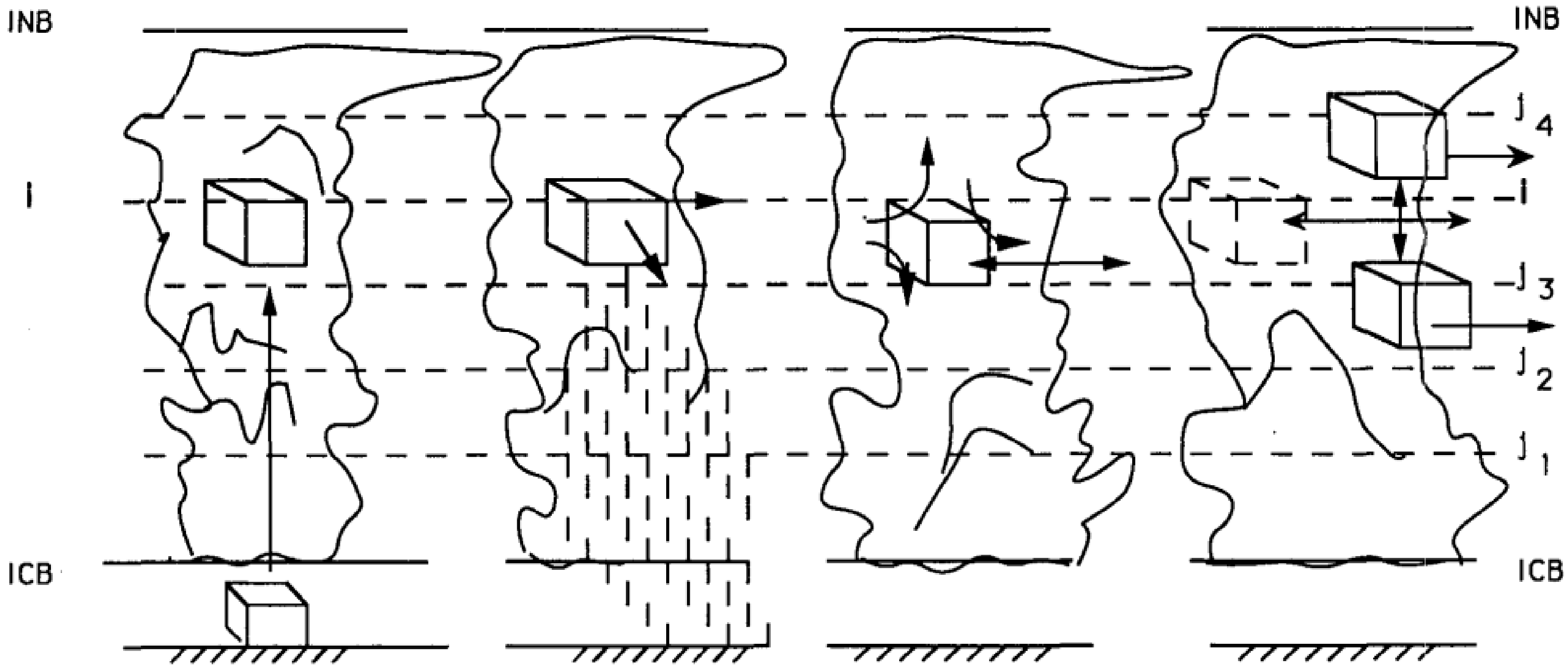


Figure 1. Schematic picture illustrating the representation of convection in the Emanuel scheme.

Bony et al., JGR, 2008



Emanuel, JAS, 1991

Step 1: perform undiluted ascent from subcloud layer to any level i between cloud base and level of neutral buoyancy

Step 2: precipitate an amount $\varepsilon_p(i)$ of the condensate that adds ta a unique downdraft

Step 3: remaining cloudy air is mixed randomly with environment at each level

Step 4: mixtures then ascent or descent to levels where hvil equals that of the environment, an amount $\varepsilon_p(j)$ of the condensate is removed for ascending mixtures

Iterate steps 3 and 4 and close with sub-cloud mass flux.