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Basic of --small-scales statistical properties in--Turbulence

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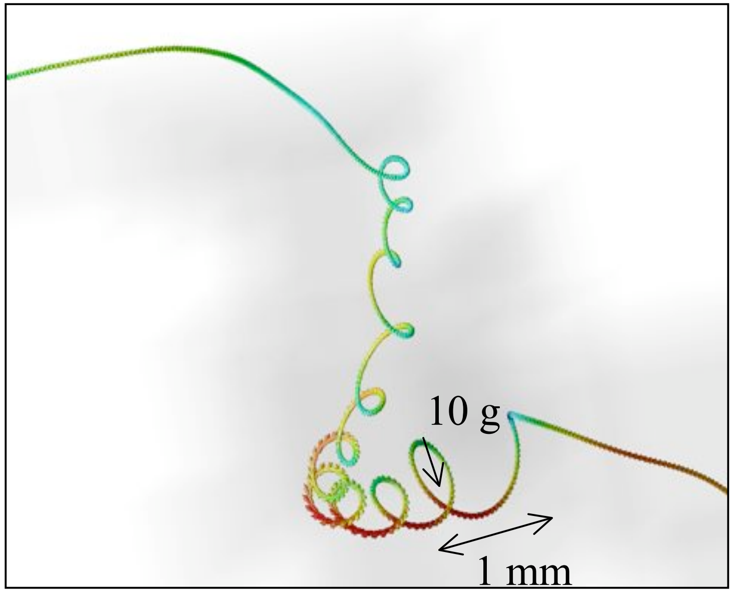


•**A successful story**: how to describe the appearance of small-scales non-Gaussian statistics using a simple phenomenology based on stochastic cascade models: large deviations theory, multifractal measures, multiaffine functions and all that.... (still: a few problems if looking at high quality numerical and experimental data)

•**A less successful story**: how to include statistics of (inertial) particles advected by the flow: the problem of preferential concentration and of the inclusions of “topological” properties in the stochastic modelization/

Leonardo da Vinci (~ 1500): “doue laturbolenza dellacqua sigenera; doue la turbolenza dellaqa simantiene plugho; doue laturbolenza dellaqau siposa”

R.P. Feynman (1970): “Certainly. I’ve spent years trying to solve some difficult problems without success. The theory of turbulence is one. In fact, it is still unsolved.”

J. Von Neumann (1949) “[…] The entire experience with the subject indicates that the purely analytical approach is beset with difficulties, which at the moment are prohibitive. […] Under these conditions there may be some hope to “break the deadlock” by extensive, but well-planned computational efforts.

Sir H. Lamb (1932): “I am an old man now, and when I die and go to Heaven there are two matters on which I hope enlightenment. One is quantum electrodynamics (QED) and the other is turbulence of fluids. About the former, I am really rather optimistic.”

- Turbulence or Turbulences?
- Why still unsolved/unsolvable: the problem of strongly non-Gaussian small-scales fluctuations.
- Large Deviations Theory & Stochastic models for 3d Homogeneous and Isotropic Turbulence (HIT).
- Toward real world (I): effects of viscosity.
- Toward real world (II): anisotropy.
- Toward real world (III): Turbulence + passive particles (tracers, heavy, light).

Occam's razor: "entia non sunt multiplicanda praeter necessitatem "
(entities must not be multiplied beyond necessity).

COMPLEX PHYSICS WITH COMPLEX FLOWS

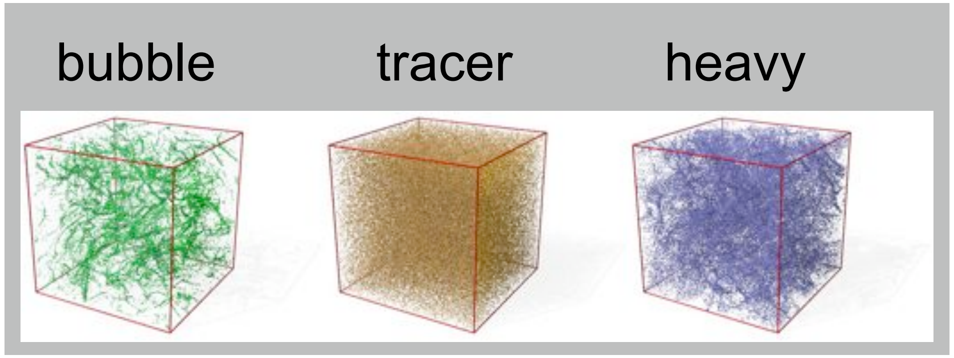
$$
\begin{cases}
\partial_t v + v \cdot \partial v = -\partial P + \nu \partial^2 v \; + F(\boldsymbol{B},\boldsymbol{B}) \; + \boldsymbol{g}\theta \; + \sum_i c_0(\boldsymbol{u}_i, \boldsymbol{v})\delta(\boldsymbol{r} - \boldsymbol{r}_i) \; + f \\
\partial_t \theta + \boldsymbol{v} \cdot \boldsymbol{\partial} \theta = \chi \partial^2 \theta \\
\partial_t \boldsymbol{B} + \boldsymbol{v} \cdot \boldsymbol{\partial B} = \boldsymbol{B} \cdot \boldsymbol{\partial v} + \chi \partial^2 \boldsymbol{B} \\
\Delta P = -\partial_i \partial_j v_i v_j \\
+ \text{ boundary conditions}
\end{cases}
$$

temperature

magnetic field

$$
\begin{cases}
\frac{d\boldsymbol{u}_i(\boldsymbol{r}_i, t)}{dt} = -\rho_f |\boldsymbol{u}_i - \boldsymbol{v}|(\boldsymbol{u}_i - \boldsymbol{v}) \\
+\rho_f \left(\frac{D\mathbf{v}}{Dt} - \frac{D\boldsymbol{u}_i}{Dt}\right) + (\boldsymbol{u}_i - \boldsymbol{v}) \times \boldsymbol{\omega}
\end{cases}
$$

small particles: drag, added mass, lift force, etc...



Flows with additives:

Advection-diffusion-reaction of passive scalar/vectors (temperature, magnetic field, chemical reactions, etc...).

Advection-diffusion of active scalars/vectors (convection, magnetic dinamo).

Polymers (drag reduction)

Bubbles/Droplets (two phase flows, rain formation, etc...)

Swimmers (cooperative hydrodynamical interactions)

COMPLEX PHYSICS WITH SIMPLE FLOWS

$$\begin{cases} \partial_t v + v \cdot \partial v = -\partial P + \nu \partial^2 v \\ \Delta P = -\partial_i \partial_j v_i v_j \\ \text{+ periodic boundary conditions} \end{cases}$$

$+f$

- homogeneous
- isotropic
- Gaussian
- white-noise in time
- large-scale

3D CASE: MAINLY UNSOLVED!

COMPLEX PHYSICS WITH SIMPLE FLOWS

$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\partial\hat{v} = -\hat{\partial}\hat{P} + \frac{1}{Re}\hat{\partial}^2\hat{v}$$

$$\begin{cases} \hat{t} = t/t_0 \\ \hat{x} = x/l_0 \\ \hat{v} = v/v_0 \end{cases}$$

$$Re \sim \frac{v\partial v}{\nu\partial^2 v} \qquad Re = \frac{l_0 v_0}{\nu}$$

Reynolds number ~ (Non-Linear)/(Linear terms)

$$Re \to \infty$$

•Fully Developed Turbulence:

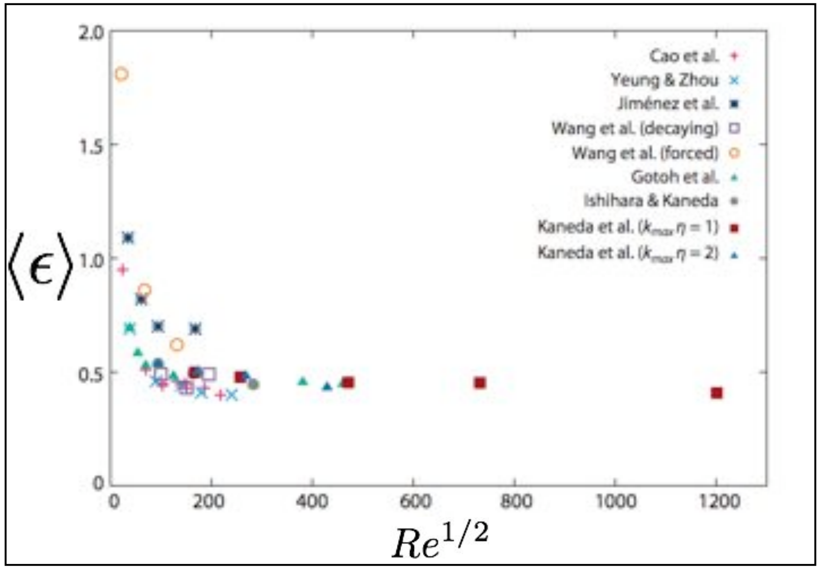
1. Strongly non-linear & non-perturbative system

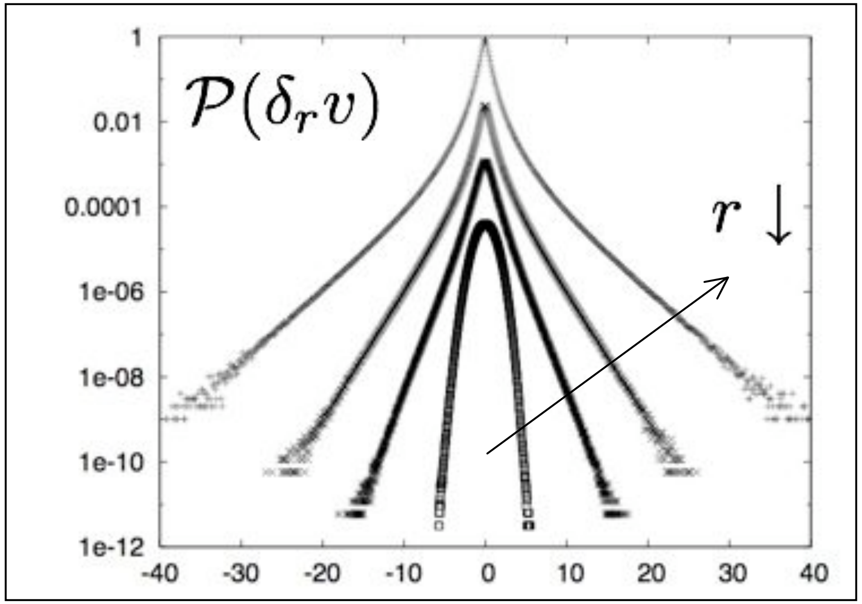
COMPLEX PHYSICS WITH SIMPLE FLOWS

$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{P} + \frac{1}{Re}\hat{\partial}^2\hat{v}$$

$$Re \to \infty$$

$$\langle\epsilon\rangle = \nu\langle(\partial v)^2\rangle \propto \frac{1}{Re}\langle(\tilde{\partial} v)^2\rangle \to const.$$





$$\delta_r v / \sqrt{\langle(\delta_r v)^2\rangle}$$

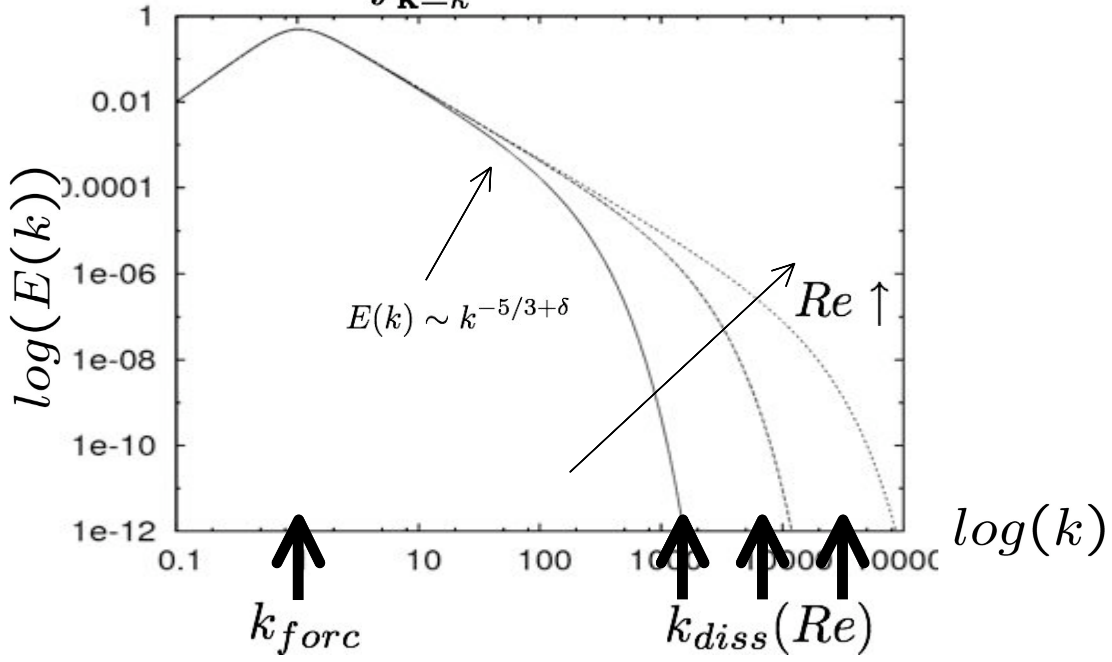
2. Out of Equilibrium (non perturbative)
•Dissipative anomaly

3. Small-scales PDFs strongly non-Gaussian
•Anomalous scaling

COMPLEX PHYSICS WITH SIMPLE FLOWS

THE ENERGY CASCADE:

$$E(k) = \int_{\mathbf{k}=k} d^3k \langle v_i(\mathbf{k}) v_i(-\mathbf{k}) \rangle$$

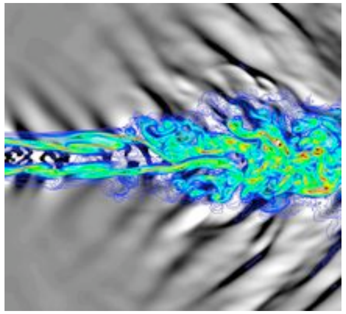


$$k_{forc} \ll k \ll k_{diss}(Re)$$

1. inertial range of scales: power law (anomalous)
2. extension increases with Reynolds!

4. Many-body problem: $\#_{dof} = (\frac{k_{diss}}{k_{forc}})^3 \sim Re^{9/4}$

Numbers.



laboratory flow

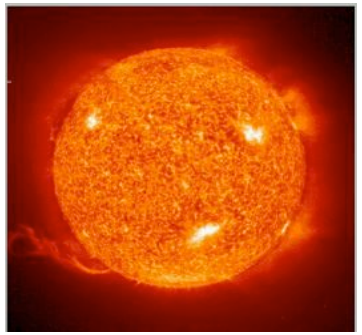
$Re \sim 10^5 - 10^9$

$\#_{dof} \sim 10^{11} - 10^{20}$

atmosph. flow

$Re \sim 10^8 - 10^{12}$

$\#_{dof} \sim 10^{18} - 10^{30}$



astrophys. flow

$Re > 10^{15}$

$\#_{dof} \sim \infty$

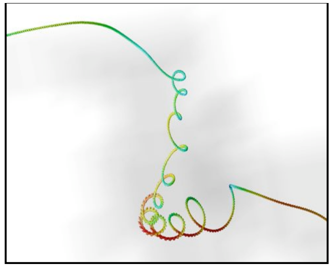
state-of-the-art DNS:

Isotropic, homogeneous Fully Periodic Flows
Pseudo-Spectral Methods.
Resolution 4096x4096x4096 (Earth Sim.)

Reynolds : 10^6,
Storage of 1 velocity configuration (float): 1 Tbyte
RAM requirements for time marching: 10 Tbyte

Moral: easy to saturate any computing power
(present and/or future)

HOW TO PROBE INTENSE FLUCTUATIONS (TAILS)?



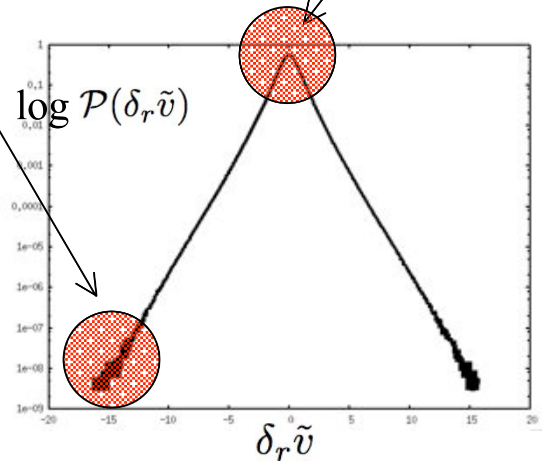
HOW TO PROBE WEAK FLUCTUATIONS (PEAKS)?

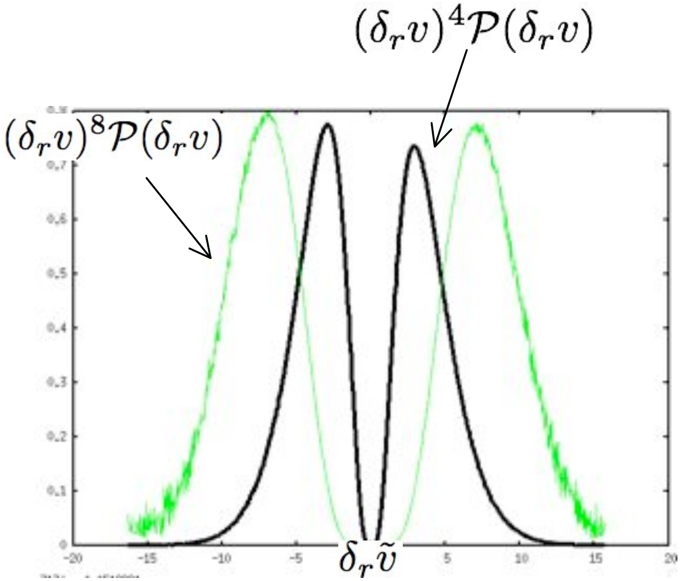
LOOK AT HIGH ORDER MOMENTS
-STRUCTURE FUNCTIONS-

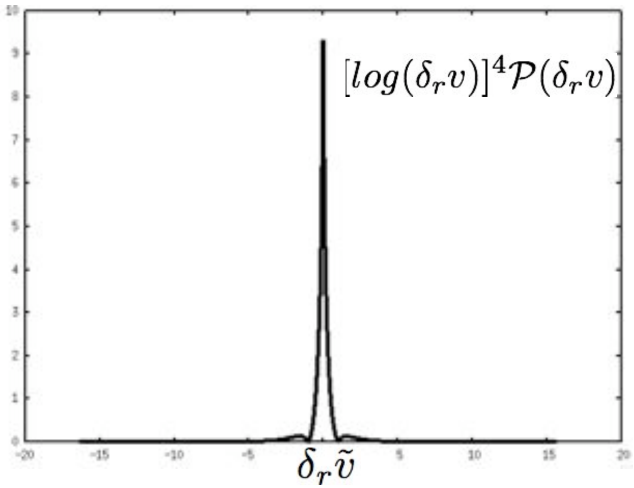
$$\langle(\delta_r v)^p\rangle \qquad p \gg 1$$

LOOK AT HIGH ORDER MOMENTS OF LOGS
-CUMULANTS-

$$\langle(log|\delta_r v|)^p\rangle \qquad p \gg 1$$







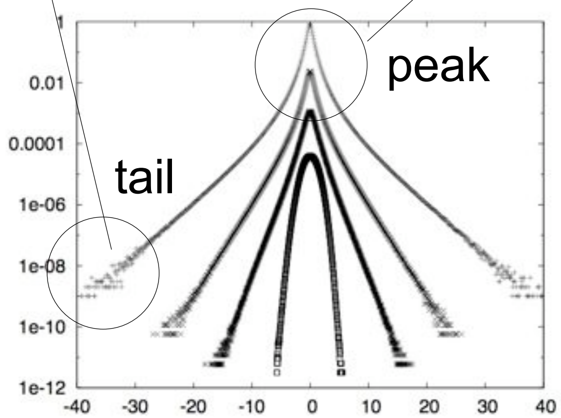
CONNECTION CUMULANTS -- STRUCTURE FUNCTIONS

$$S_p(r) = \langle (\delta_r v)^p \rangle$$

$$\kappa_n(r) = \langle (log|\delta_r v|)^n \rangle$$

$$S_p(r) = \exp \sum_n C_n(r) \frac{p^n}{n!}$$

$$C_1(r) = \kappa_1(r)$$
$$C_2(r) = \kappa_2(r) - (\kappa_1(r))^2$$
$$C_n(r) = \kappa_n(r) + f(\kappa_{n-1}...)$$

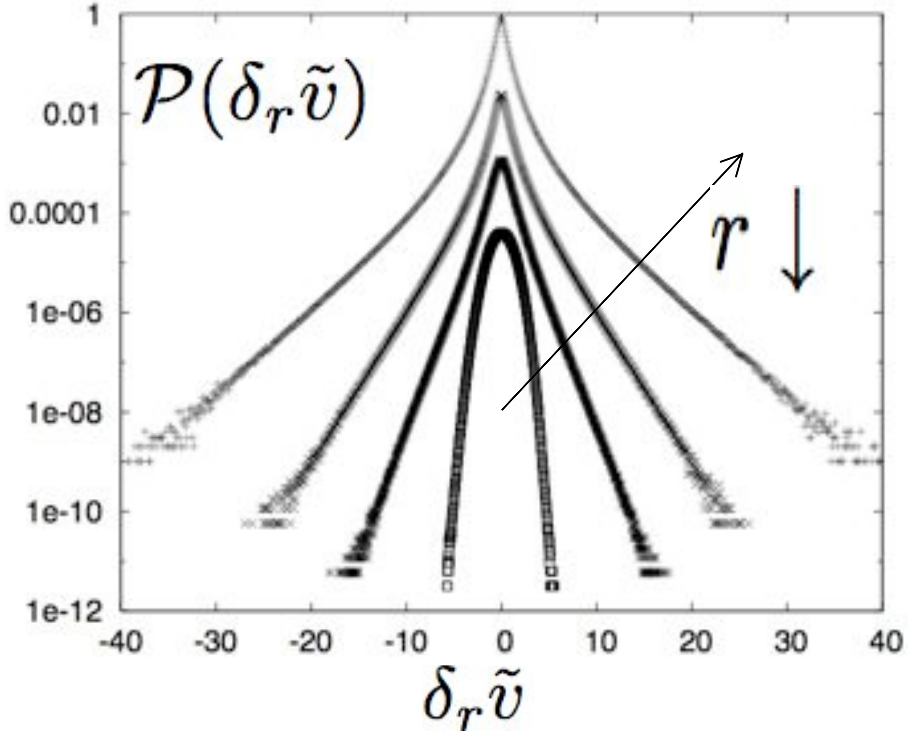


$$S_p(r) \sim r^{\zeta(p)}$$

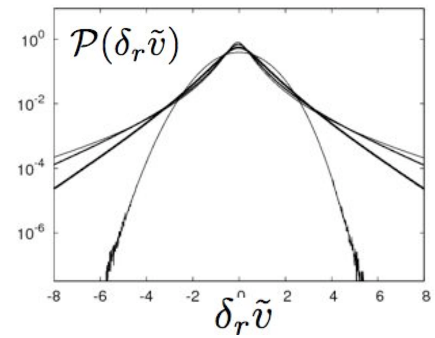
$$C_n(r) \sim c_n log(r)$$

[Delour Muzy Arneodo EPJB 2001]

INTERMITTENCY

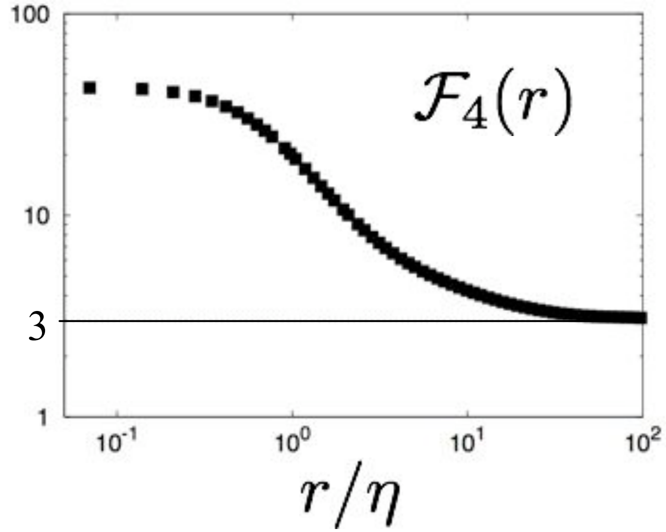


$$\delta_r \tilde{v} = \frac{\delta_r v}{\langle (\delta_r v)^2 \rangle^{1/2}}$$



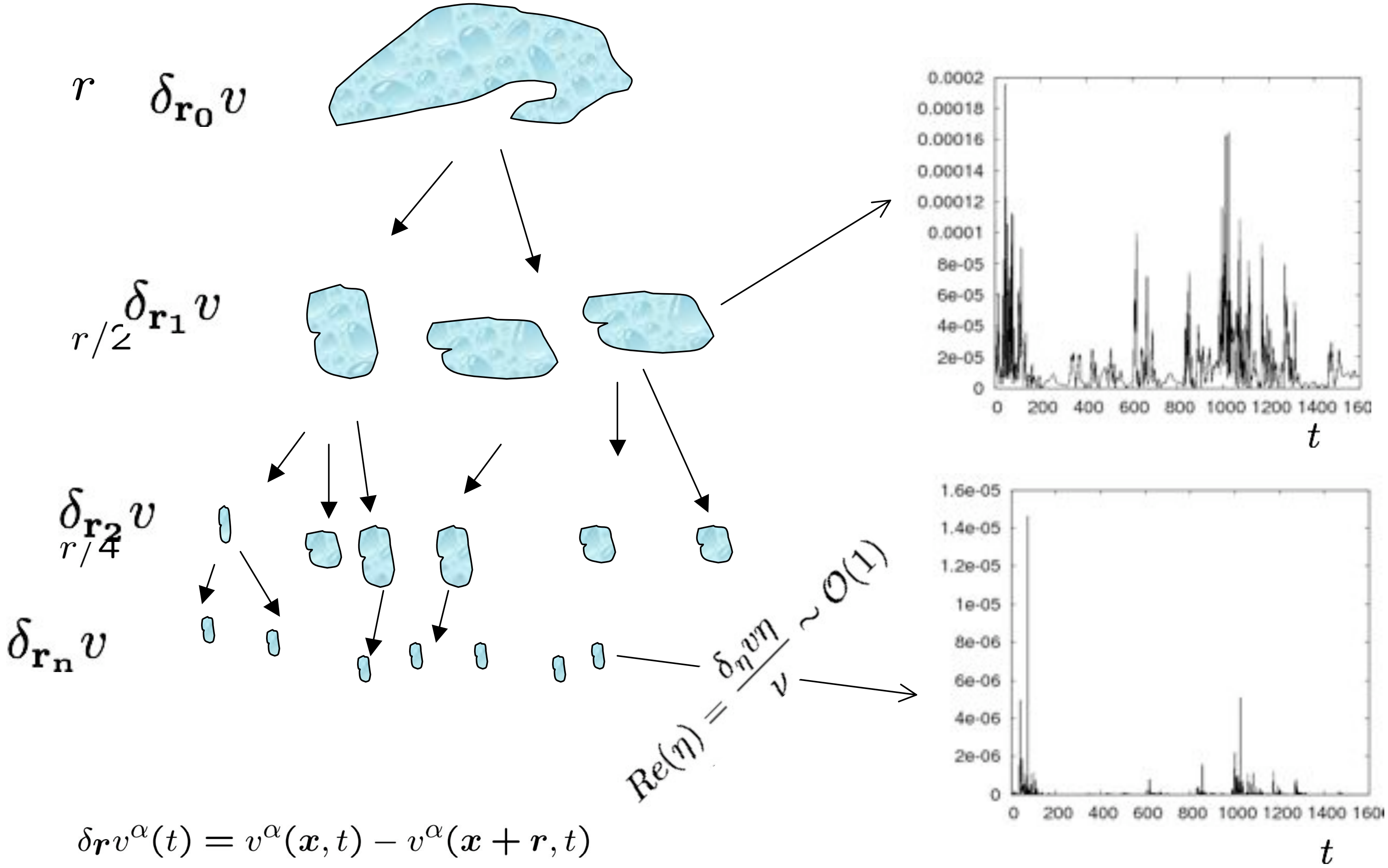
$$\mathcal{F}_{2p}(r) = \langle (\delta_r \tilde{v})^{2p} \rangle = \frac{\langle (\delta_r v)^{2p} \rangle}{\langle (\delta_r v)^2 \rangle^p}$$

$$\mathcal{F}_{2p}(r) \neq const.$$



spatio-temporal Richardson cascade

$$Re(r) = \frac{r\delta_r v}{\nu}$$



$$\delta_{\boldsymbol{r}} v^\alpha(t) = v^\alpha(\boldsymbol{x}, t) - v^\alpha(\boldsymbol{x} + \boldsymbol{r}, t)$$

$$S_n^{\bar{\alpha}}(\bar{\boldsymbol{r}}, \bar{t}) = \langle \delta_{\boldsymbol{r}_1} v^{\alpha_1}(t_1) \cdots \delta_{\boldsymbol{r}_n} v^{\alpha_n}(t_n) \rangle$$

energy transfer in 3d turbulence

what do we know from analytical results

$$\partial_t \boldsymbol{v} + \boldsymbol{v}\cdot\boldsymbol{\partial}\boldsymbol{v} = -\boldsymbol{\partial} P + \nu\cancel{\partial^2} \boldsymbol{v} + \cancel{\boldsymbol{f}}$$

$$\boldsymbol{v}' \to \lambda^h \boldsymbol{v}$$
$$\boldsymbol{x}' \to \lambda \boldsymbol{x}$$
$$t' \to \lambda^{1-h} t$$

$$\forall h \qquad \qquad \eta \ll r \ll L_0$$

Scaling invariance in the Inertial Range

Third order longitudinal structure functions:

$$S_3(r) = \langle (\hat{\boldsymbol{r}} \cdot \delta_{\boldsymbol{r}} \boldsymbol{v})^3 \rangle$$

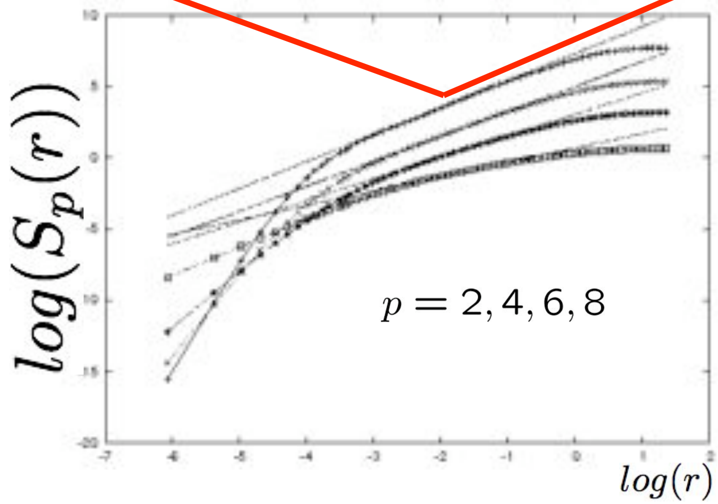
$$\to h = \frac{1}{3}$$

Howart-von Karman: EXACT FROM NAVIER-STOKES EQS.

$$\delta_r v \sim r^h \qquad \qquad E(k) \sim k^{-2h-1}$$

K41

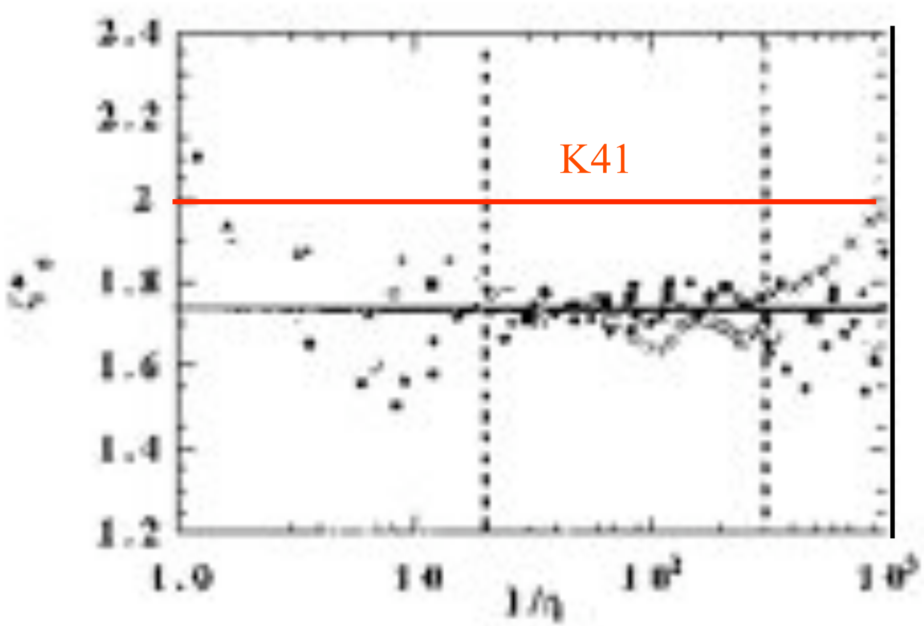
$$\zeta_p(r) \stackrel{\text{DEF}}{=} \frac{d\,log(S_p(r))}{d\,log(r)}$$



SCALE-BY-SCALE MAGNIFYING GLASS

in log-log all cows are black!

$$\eta \ll r \ll l_0$$

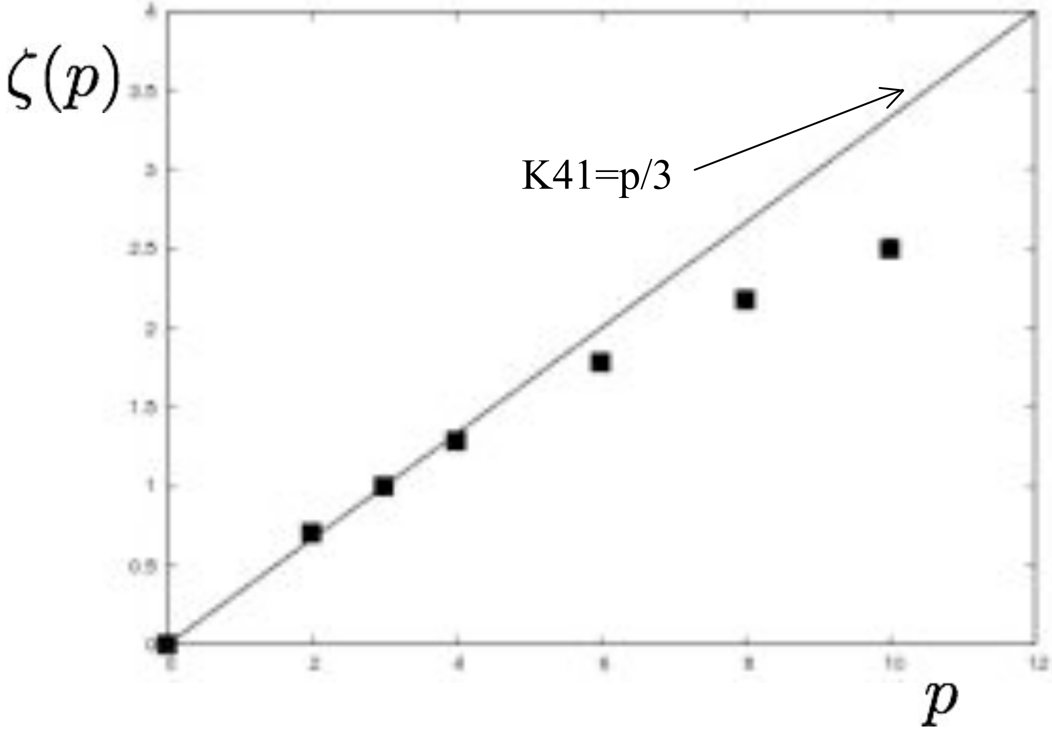


$$\frac{d\,log(S_p(r))}{d\,log(r)} = \frac{p}{3}$$

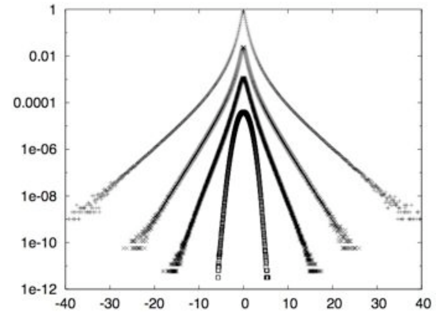
Exp	Configuration	Λ	η	R_λ	u'/U (%)	l_w/η	f_s/f_η	Ref
1	swirling flow	10 cm	25-50 μm	200-5000	20-40	0.1-3	0.5-5	[2]
2a	jet	20 cm	0.28 mm	428	25	2.5	7	[3]
2b	wind tunnel	10 cm	0.35 mm	3050	7	1.2	3	
3	jet	1 cm	7 μm	580	25	3	7	[4]
4a	cylinder	5-10 cm	0.2-0.5 mm	100-300	15	1-2.5	7	[5]
4b	jet	10 cm	0.1 mm	800	30	5	7	
5a	jet	7.5 cm	0.095 mm	810	18	2	1	[6]
5b	grid	17 cm	0.19 mm	530	8	1	1	
6	jet	4-8 cm	22-28 μm	240-330	20-25	0.6-1.3	–	[7]
7	grid	4 mm-1 cm	100-250 μm	35-110	1.5-8	3-10	1-3	[8]

A. Arneodo *et al.*, Europhys. Lett. **34**, 411 (1996).

$$\sim r^{\zeta(p)}$$



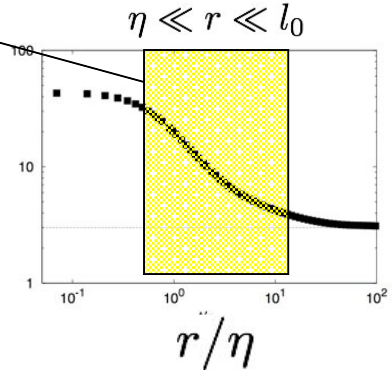
$$\epsilon^{p/3} \neq \langle \epsilon(r)^{p/3} \rangle$$



$$\delta_r \tilde{v} = \frac{\delta_r v}{\langle(\delta_r v)^2\rangle^{1/2}}$$

$$F_4(r) = \frac{S_4(r)}{(S_2(r))^2} \sim r^{\zeta(4)-2\zeta(2)}$$

$$F_6(r) = \frac{S_6(r)}{(S_2(r))^3} \sim r^{\zeta(6)-3\zeta(2)}$$



1. small-scales are intermittent (neq k41)
2. power-law behaviour in the inertial range

↓

Eulerian Multifractal Formalism
and
Large Deviations Theory

The “Standard Model” at Re$= \infty$

$$S_p(r) = \langle [\boldsymbol{v}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{v}(\boldsymbol{x})]^p \rangle \qquad \eta \ll r \ll L_0$$

$$\delta_r v \sim v_0 (\frac{r}{L_0})^h \quad \mathcal{P}_h(r) \sim (\frac{r}{L_0})^{3-D(h)}$$

$$S_p(r) = \langle (\delta_r v)^p \rangle \sim \langle v_0^p \rangle \int_I dh \left(\frac{r}{L_0} \right)^{hp+3-D(h)}$$

$$S_p(r) \sim \left(\frac{r}{L_0} \right)^{\zeta_p}$$

Parisi-Frisch 1983

$$\zeta_p = \inf_h (hp + 3 - D(h))$$

$$\mathcal{F}_{2p}(r) = r^{\zeta(2p) - p\zeta(2)}$$

$D(h)$

What about PDFs?

$$\delta_r v \sim v_0 (\frac{r}{L})^h$$

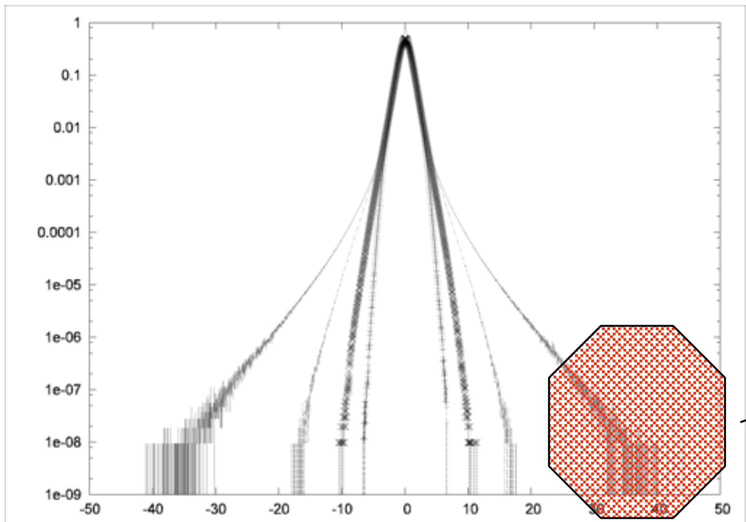
Experimental results tell us PDF at large scale is close to Gaussian

$$\mathcal{P}(v_0) \sim \exp(v_0^2/2)$$

$$\mathcal{P}(\delta_r v) \sim \int dh dv_0 \mathcal{P}(v_0) \mathcal{P}_r(h)$$

$$P(\delta_r v) \sim \int dh \, (\tfrac{r}{L})^{3-h-D(h)} \, \mathrm{exp}(-\frac{(\delta_r v)^2}{2(r/L)^{2h}})$$

Superposition of Gaussians with different width:



$P(\delta_r v)$

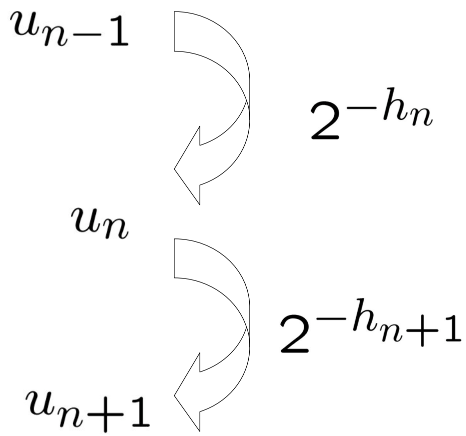
$\delta_r v / \langle (\delta_r v)^2 \rangle^{1/2}$

How to derive D(h) from the equation of motion?

Physical intuition of D(h): the result of a random energy cascade

$$r_n = 2^{-n} L$$

$$u_n = \delta_{r_n} v$$



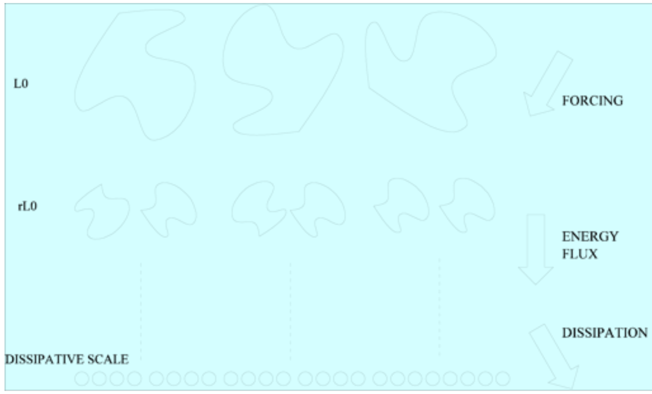
$$u_n \sim a(n, n-1) u_{n-1}$$

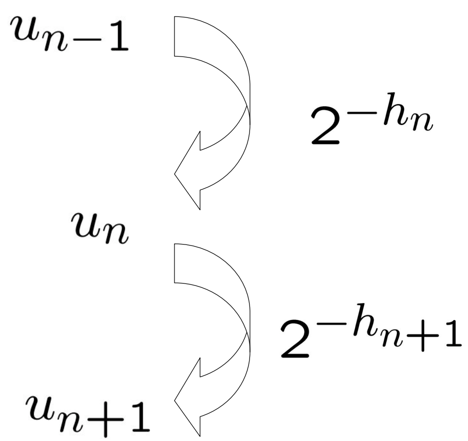
$$u_n = 2^{-h_n} u_{n-1} \qquad h_n = -log_2(u_n / u_{n-1})$$

locality of interactions among neighboring scales $\longrightarrow$
multipliers almost uncorrelated $\longrightarrow$ small scales universality

$$r_n = 2^{-n}L$$

$$u_n = \delta_{r_n} v$$





Large deviation theory

$$u_n = (\Pi_{i=1}^{n} 2^{-h_i}) u_0 \equiv 2^{-n(\frac{1}{n}\sum_{i=1}^{n} h_i)} u_0 \qquad u_n \sim \left(\frac{r_n}{L}\right)^h u_0$$

$$\langle u_n^p \rangle \sim u_0^p \int dh \left(\frac{r_n}{L}\right)^{hp+S(h)}$$

★ **! Scaling is recovered in a statistical sense, no local scaling properties !** ★

$$n \rightarrow \infty$$

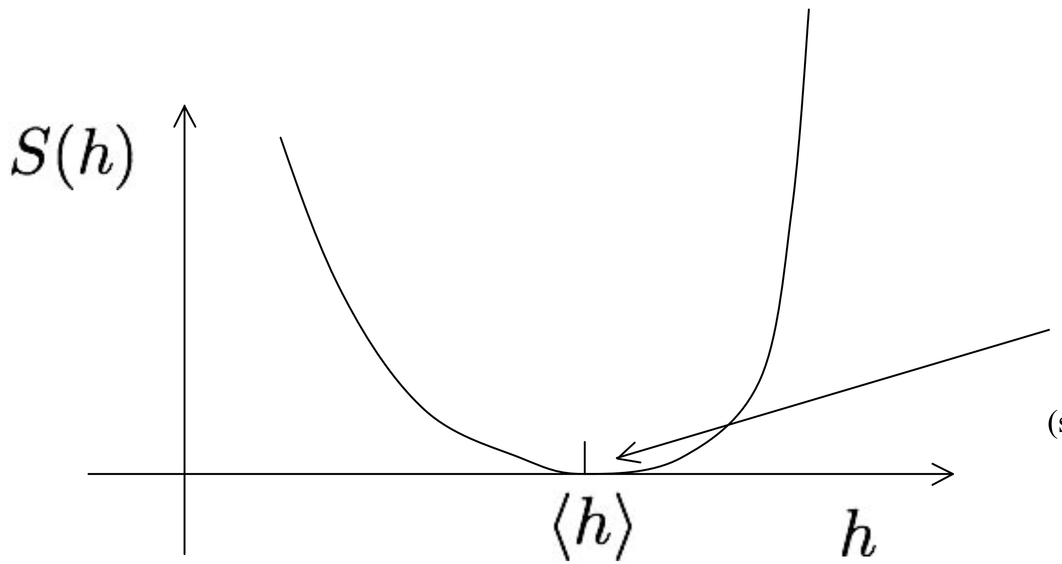
$S(h)$ [Cramer function]: non-negative definined, with a minimum for

$$S(h = \langle h \rangle) = 0$$

large number law

(sum- of n independent random variables tends to its mean value with probability one, when n -> infty)

$$\langle h \rangle = lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0,n} h_i$$



$$S(h) \sim (h - \langle h \rangle)^2$$

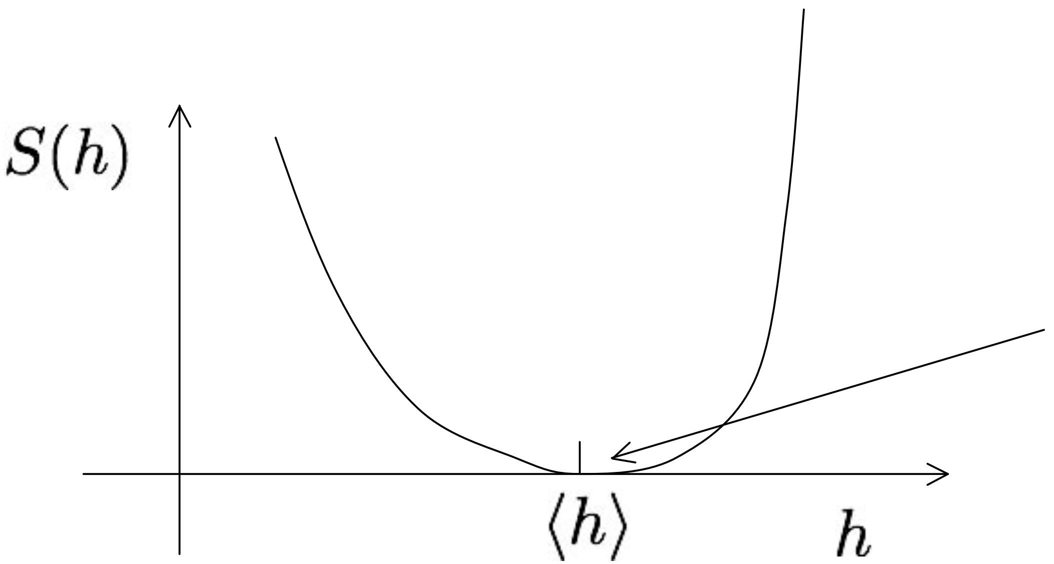
central limit theorem

(small deviations from the mean are normally distributed)

•ONLY SMALL DEVIATIONS AROUND THE MEAN ARE NORMALLY DISTRIBUTED:

NO REASON FOR S(h) TO BE QUADRATIC FOR ALL h!!!! : NO REASON FOR THE PROBABILITY DISTRIBUTION FUNCTION OF $\delta_r v$ TO BE LOG-NORMAL

$$P(h) \sim (\frac{r_n}{L})^{S(h)}$$



$$S(h) \sim (h - \langle h \rangle)^2$$

central limit theorem

(deviations from the mean are normally distributed)

Cramer function as the characteristic functions of random multipliers

$$Z(\beta) = \langle e^{-\beta h} \rangle$$

if i.i.d.

$$Z^n(\beta) = \langle e^{-\beta \sum_{i=1,n} h_i} \rangle$$

$$\langle e^{-\beta \sum_{i=1,n} h_i} \rangle = \int dh e^{-n(\beta h - S(h))}$$

$$Z^n(\beta) \sim e^{n \max_h [S(h) - \beta h]}$$

$$\begin{cases} log Z(\beta) = \max_h [S(h) - \beta h] \\ S(h) = \min_\beta [log Z(\beta) + \beta h] \end{cases}$$

example:	$S(h) = -h log(h) - (1-h) log(1-h) - log(2)$
coin tossing	$0 \le h \le 1$

$$u_n \sim (\frac{r_n}{L})^h u_0 \qquad P_{r_n}(h) \sim (\frac{r_n}{L})^{S(h)}$$

$$\langle u_n^p \rangle = \int dh P_{r_n}(h) u_n^p$$

$$\langle u_n^p \rangle \sim u_0^p \int dh (\frac{r_n}{L})^{hp+S(h)}$$

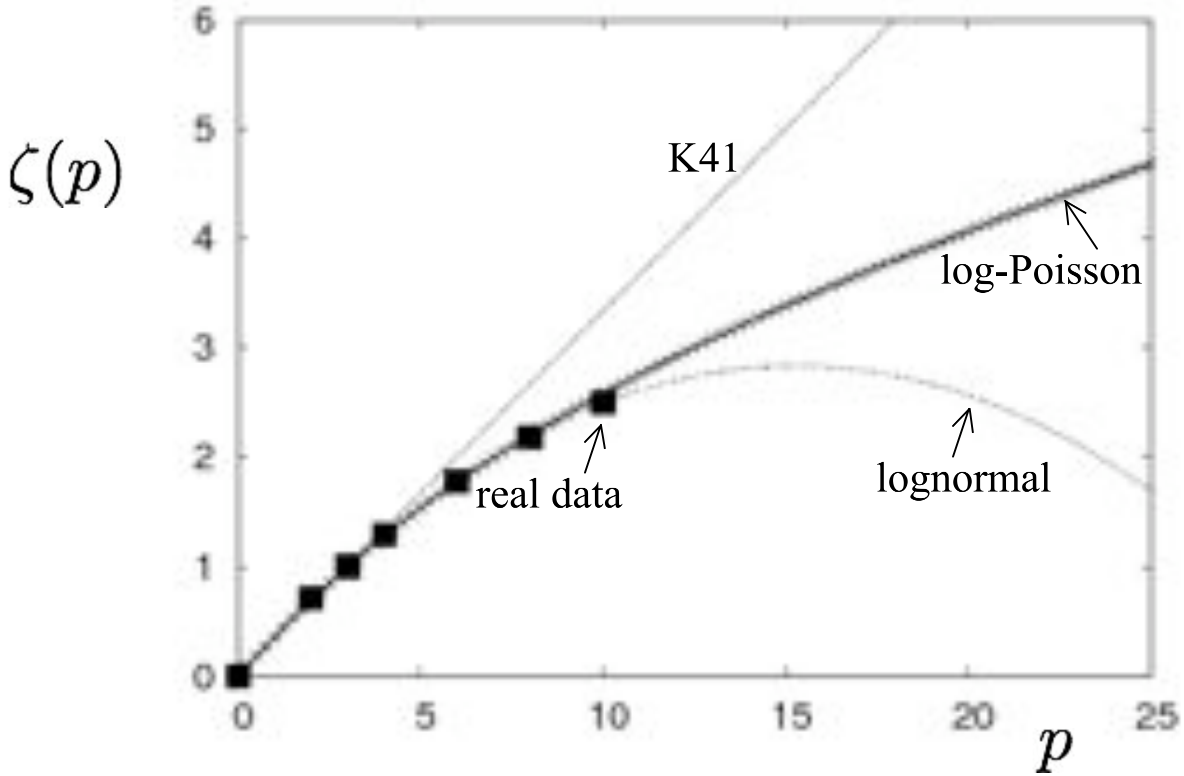
•intermittency: beating between power-law observables and power law distribution functions

$$\langle u_n^p \rangle = \int dh P(h) u_n^p \sim (\frac{r_n}{L})^{\zeta(p)}$$

$$\zeta(p) = \min_h (ph + S(h))$$

$$3 - D(h) = S(h)$$

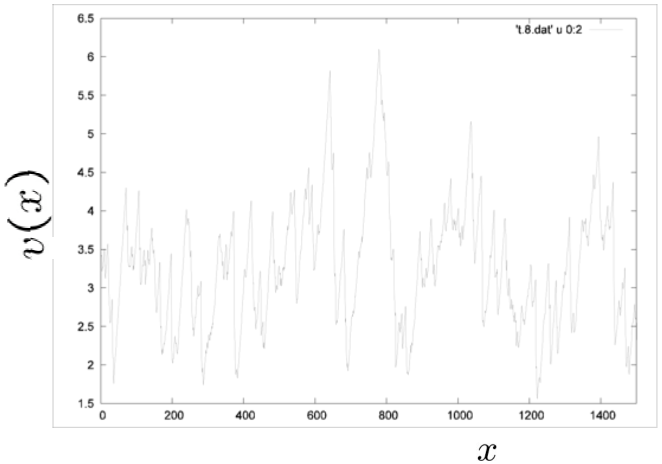
shortcomings of log-normal



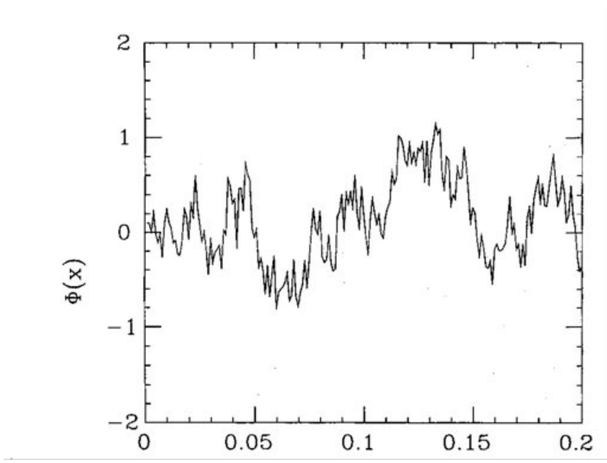
- $d\zeta(p)/dp < 0 \rightarrow \delta_r v \sim r^h \qquad h < 0$ [supersonic events!]
- violation of Novikov inequality if applied to energy dissipation statistics

Synthesis & Analysis

•How to build a multiaffine field with prescribed scaling laws
•How to distinguish synthetic and real fields

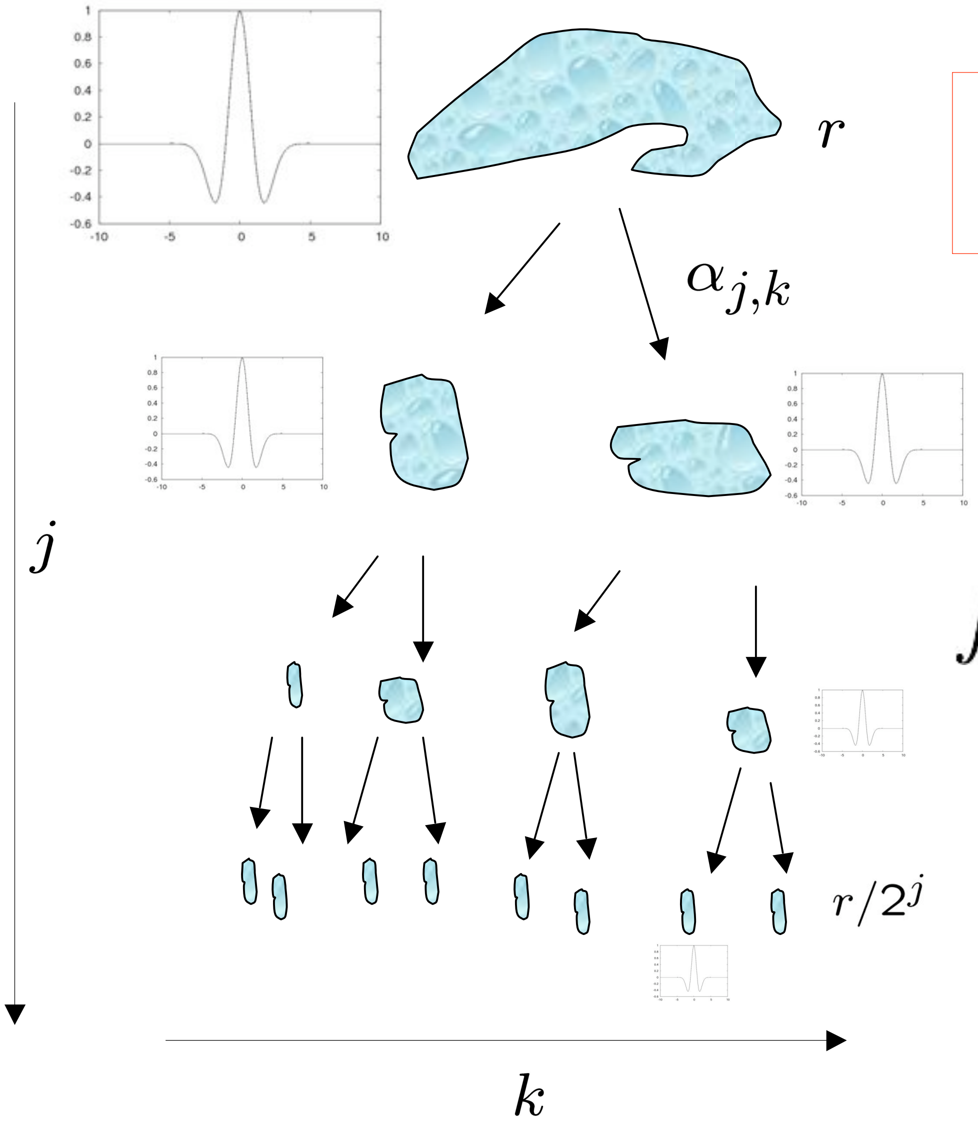


Modane wind tunnel (curtesy of Y. Gagne)



Multaffine field (wavelets based)

Richardson cascade: random multiplicative process



synthesis

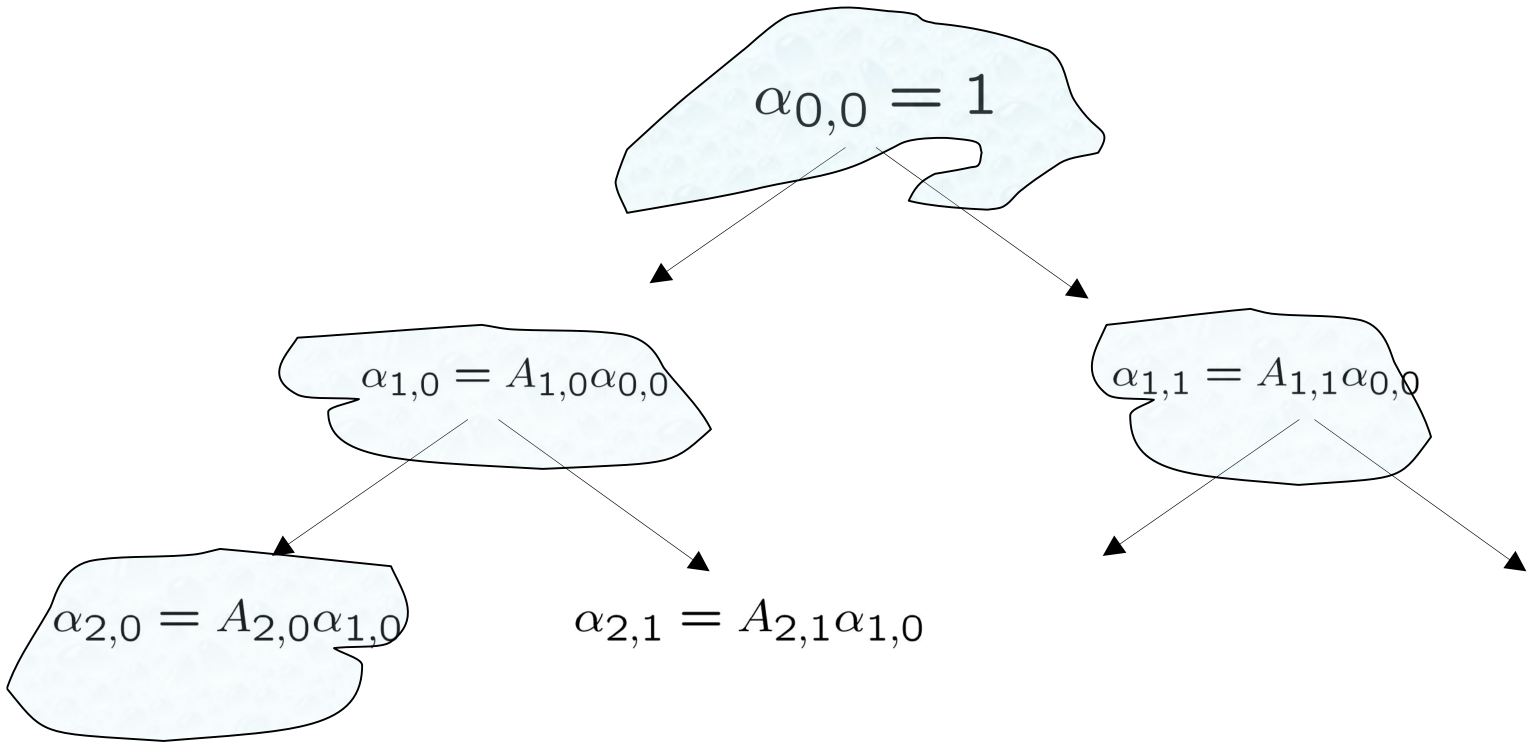
$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$$

hortonormal eigenbasis

$$\int dx \psi_{jk}(x)\psi_{j'k'}(x) = \delta_{jj'}\delta_{kk'}$$

analysis

$$\alpha_{j,k} = \int dx \psi_{j,k}(x) v(x)$$



Multiplicative uncorrelated structure

$$\langle|\alpha_{j,k}|^p\rangle = \langle A^p\rangle\langle|\alpha_{j-1,k}|^p\rangle = 2^{j\log_2(\langle A^p\rangle)}\langle|\alpha_{0,0}|^p\rangle$$

$$\langle|v(x+r) - v(x)|^p\rangle \sim r^{\xi(p)}$$

$$\xi(p) = -\frac{1}{2}p - \log_2(\langle A^p\rangle)$$

$$S_2(r) = \langle \Sigma_{j,k} (\alpha_{j,k} 2^{j/2} (\psi(2^j x + 2^j r - k) - \psi(2^j x - k)))^2 \rangle$$

+ Spatial Ergodicity

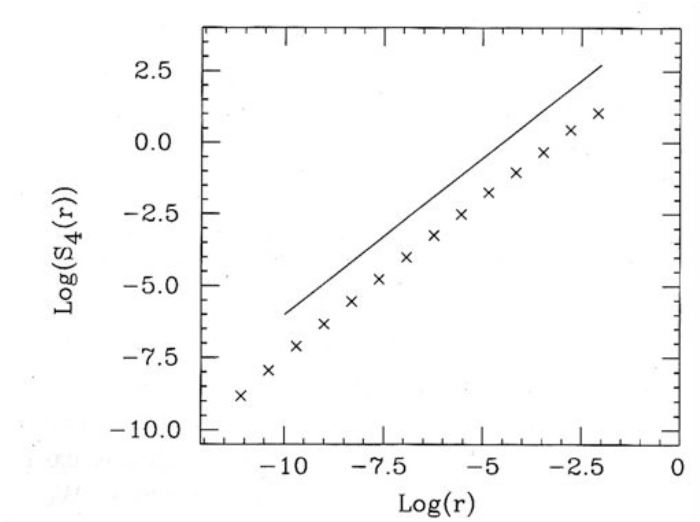
$$S_2(r) = \Sigma_{j,k} 2^j \langle \alpha_{j,k}^2 \rangle \langle (\psi(2^j x + 2^j r - k) - \psi(2^j x - k))^2 \rangle$$

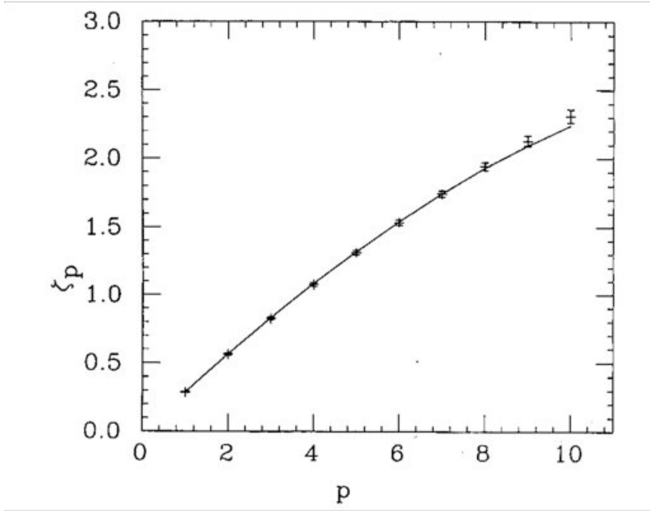
$$G_2(r) = \int dx (\psi(x + r) - \psi(x))^2 \qquad S_2(r) = \Sigma_j 2^j \langle \alpha_{j,k}^2 \rangle G_2(2^j r)$$

$$S_2(2r) = \Sigma_j 2^j \langle \alpha_{j,k}^2 \rangle G_2(2^{j+1} r) = \Sigma_j 2^{j(1+\log_2(\langle A^2 \rangle)} G_2(2^{j+1} r)$$

$$S_2(2r) = 2^{-(1+\log_2(\langle A^2 \rangle))} \Sigma_j 2^{(j+1)(\log_2(\langle A^2 \rangle + 1))} G_2(2^{j+1} r) = 2^{-(1+\log_2(\langle A^2 \rangle))} S(r)$$

•Benzi et al. Physica D (1993) vol. 65 (4) pp. 352-358



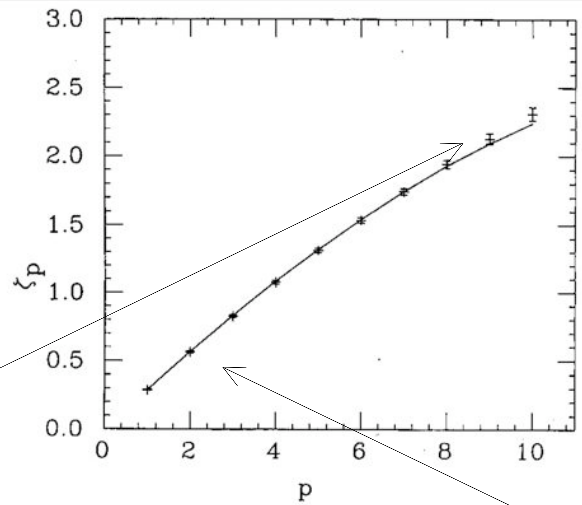


CONNECTION CUMULANTS -- STRUCTURE FUNCTIONS

$$S_p(r) = \langle (\delta_r v)^p \rangle \qquad \kappa_n(r) = \langle (log|\delta_r v|)^n \rangle$$

$$S_p(r) = \exp \sum_n C_n(r) \frac{p^n}{n!}$$

$\zeta(p)$

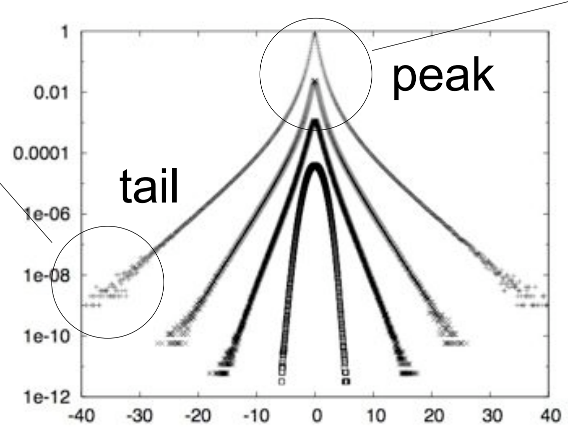


$$C_1(r) = \kappa_1(r)$$
$$C_2(r) = \kappa_2(r) - (\kappa_1(r))^2$$
$$C_n(r) = \kappa_n(r) + f(\kappa_{n-1}...)$$

$$C_n(r) \sim c_n log(r)$$

$$S_p(r) \sim \left(\frac{r}{L_0}\right)^{\zeta_p}$$

$$c_n = \frac{d^n}{dp^n}\zeta(p)|_{p=0}$$



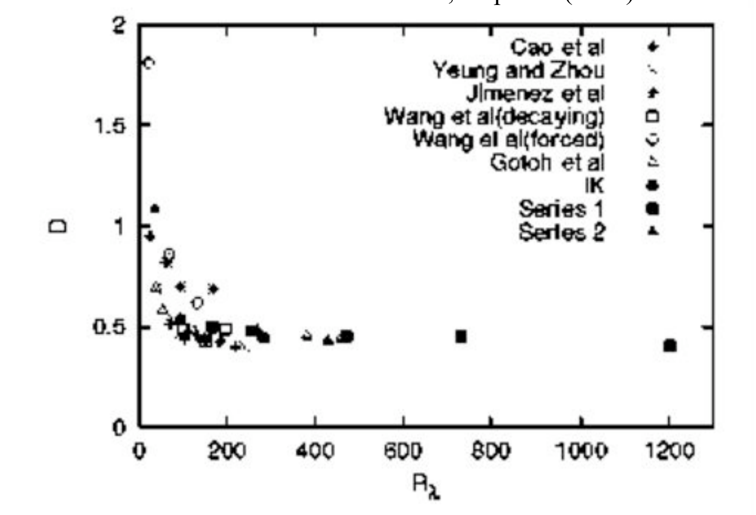
Toward real world (I): finite Reynolds effects.

Energy dissipation is Reynolds independent:
<u>Dissipative anomaly</u>

$$\lim_{Re\to\infty} \equiv \lim_{\nu\to 0}$$

$$\epsilon = \nu\langle(\boldsymbol{\partial v})^2\rangle \to const.$$

Kaneda et al PoF, 15 p L21 (2003)



How to derive the statistics of gradients within the multifractal/ldt formalism?

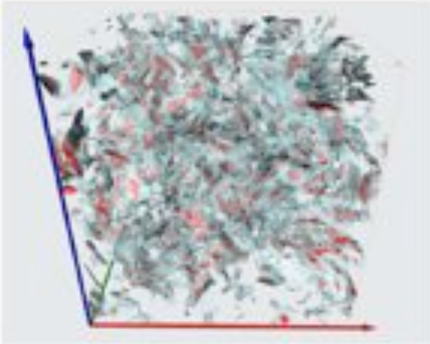
$$Re(r) = \frac{r\delta_r v}{\nu}$$

$$\boldsymbol{v}\cdot\boldsymbol{\partial v} \sim \nu\partial^2\boldsymbol{v} \longrightarrow Re(\eta) \sim O(1) \longrightarrow \frac{\eta\delta_\eta v}{\nu} \sim \mathcal{O}(1)$$

$$\delta_\eta v \sim v_0\left(\frac{\eta}{L}\right)^h \longrightarrow \eta^{1-h} \sim \nu L^h/v_L$$

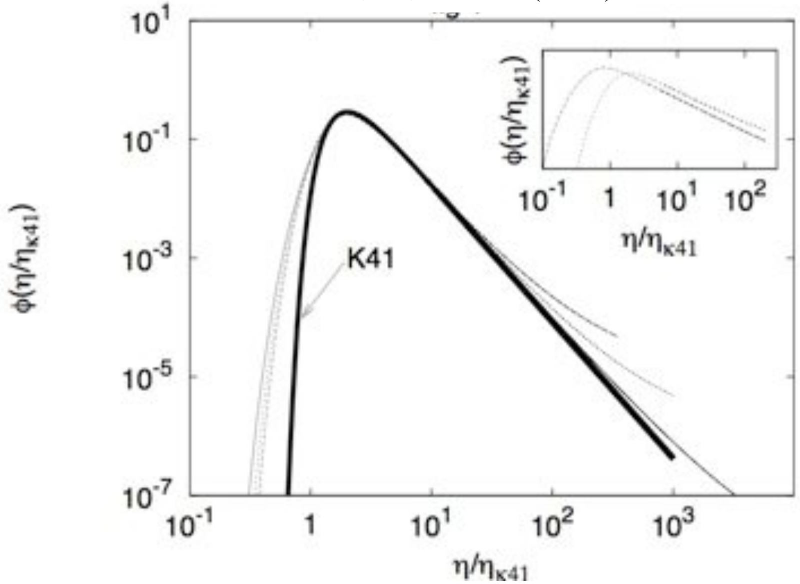
<u>Dissipative scale fluctuates</u> $\eta(h) \sim Re^{-\frac{1}{1+h}}$

J. Schumacher, EPL 80, 54001 (2007)



$$\frac{\delta_\eta v\, \eta}{\nu} \sim \mathcal{O}(1)$$

L.B. PoF , 80, 031703 (2008)



- Statistics of gradients highly non trivial

$$s = \frac{\delta_\eta v}{\eta} \qquad s = v_0 \eta^{h-1}/L^h$$

$$\mathcal{P}(s) = \int dh dv_0 \mathcal{P}(v_0) \mathcal{P}_\eta(h)$$

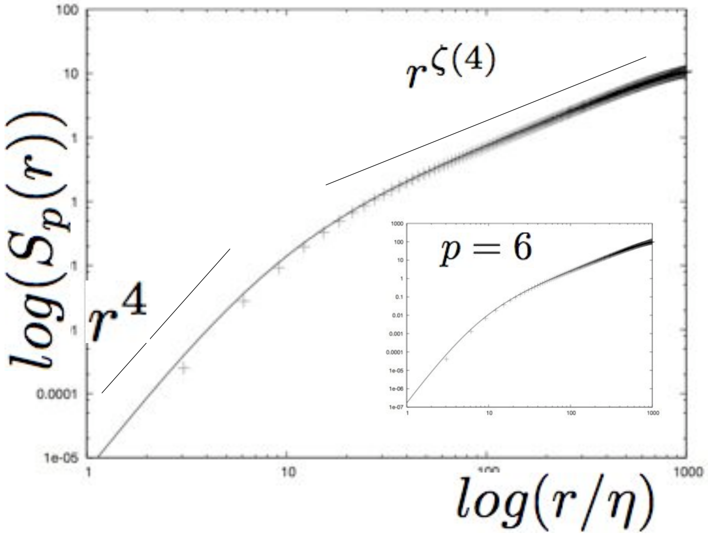
$$y(h) = \frac{4-[h+D(h)]}{2}$$

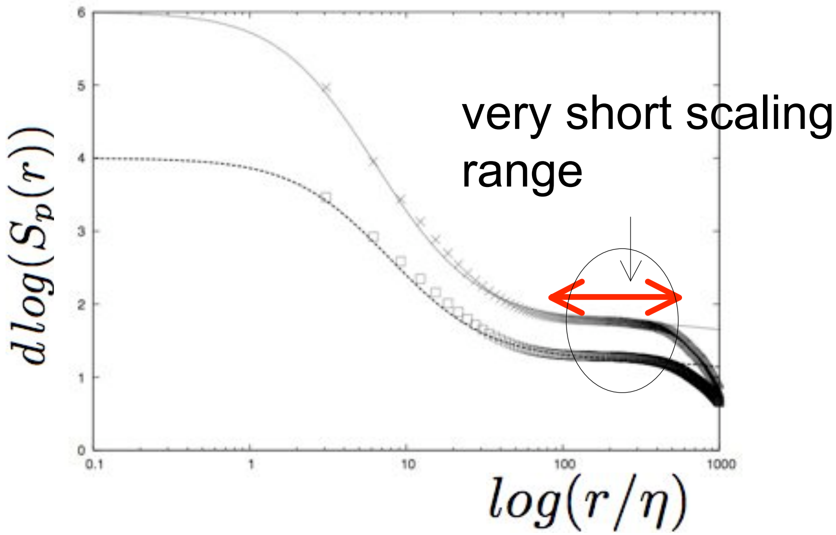
★ $\frac{s}{\langle s^2 \rangle^{1/2}} > 1$ ★

$$\langle s^p \rangle \sim Re^{\zeta(p)}$$

real world: High Resolution DNS

$p = 4$





$$S_p(r) \sim r^{\zeta(p)}$$

$$\zeta_p = \inf_h \left(hp + 3 - D(h)\right)$$

$$\frac{d\,log(S_p(r))}{d\,log(r)} \sim \zeta(p)$$

REMOVING FOCUS ON PURE POWER LAW:

TYPICALLY NEVER OBSERVED IN DNS OR CONTROLLED LABORATORY EXPERIMENTS (MODERATE REYNOLDS NUMBERS)

AT HIGH REYNOLDS NUMBERS (ABL, SOLAR WIND ETC..) CONTAMINATION FROM ANOSOTROPIES OR/AND NON-HOMOGENEITIES (DIFFICULT TO CONTROL)

IN PRESENCE OF FINITE INERTIAL RANGE EXTENSION: WHAT TO CONTROL? HOW TO TEST QUANTITATIVELY INFLUENCE/IMPORTANCE OF VISCOUS AND INTEGRAL SCALES?

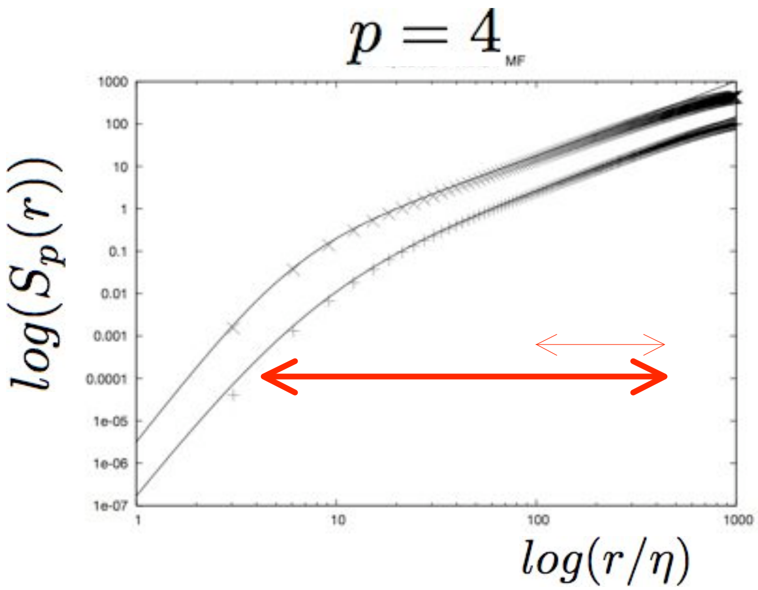
$$\eta \ll r \ll L_0$$

$$\eta \leq r \ll L_0$$

HOW TO CHECK D(h) QUANTITATIVELY CONSIDERING THE NATURAL LIMITATIONS IN THE INERTIAL RANGE EXTENSIONS?

LOOK FOR THE EFFECTS OF VISCOUS SCALES.
THE SO-CALLED: INTERMEDIATE DISSIPATIVE RANGE

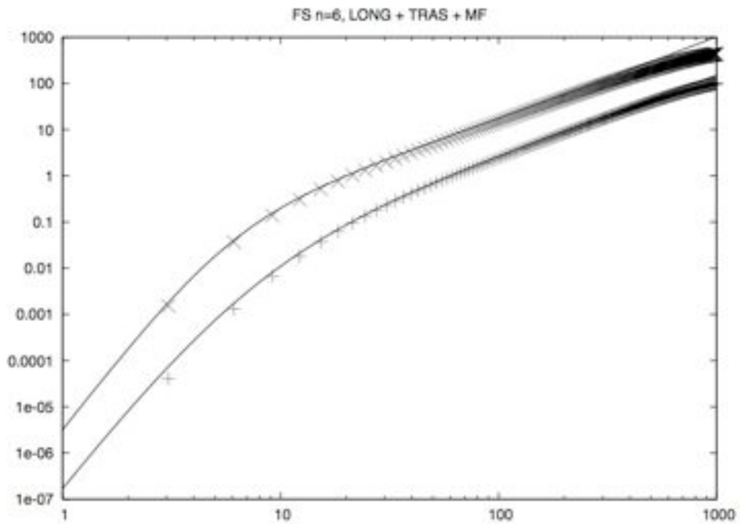
AND TRY TO TEST MULTIFRACTAL/LDT PREDICTION ALSO ON THIS EXTENDED RANGE OF SCALES



BATCHELOR-MENEVEAU PARAMETRISATION

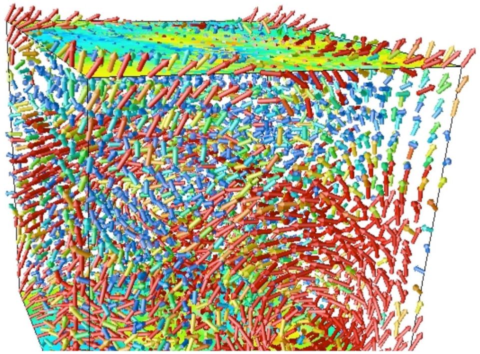
$$\delta_r v \sim v_0 \frac{r}{[(\frac{\eta(h)}{L_0})^\alpha + (\frac{r}{L_0})^\alpha]^{(1-h/)\alpha}}$$

$$\begin{cases} \delta_r v \sim r^h & \eta \ll r \ll L_0 \\ \delta_r v \sim \frac{\delta_\eta v}{\eta} r & r \ll \eta \\ \eta(h) \sim Re^{-\frac{1}{1+h}} & \end{cases}$$



α	Free parameter

EULERIAN



$$\delta_r \mathbf{u} = [\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})]$$

$$\begin{cases} \delta_r u_L = [\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})] \cdot \hat{\mathbf{r}} \\ \delta_r u_T = [\mathbf{u}(\mathbf{x}+\mathbf{x}) - \mathbf{u}(\mathbf{x})] \cdot \hat{\mathbf{n}} \end{cases}$$

$$S_{L,T}^{(p,q)}(r) = \langle (\delta_r u_L)^p (\delta_r u_T)^q \rangle$$

2nd $\begin{cases} S_{L,T}^{(2,0)}(r) = S_L^{(2)}(r) \\ S_{L,T}^{(0,2)}(r) = S_T^{(2)}(r) \end{cases}$ 3rd $\begin{cases} S_{L,T}^{(1,2)}(r) \\ S_{L,T}^{(3,0)}(r) = S_L^{(3)}(r) \end{cases}$

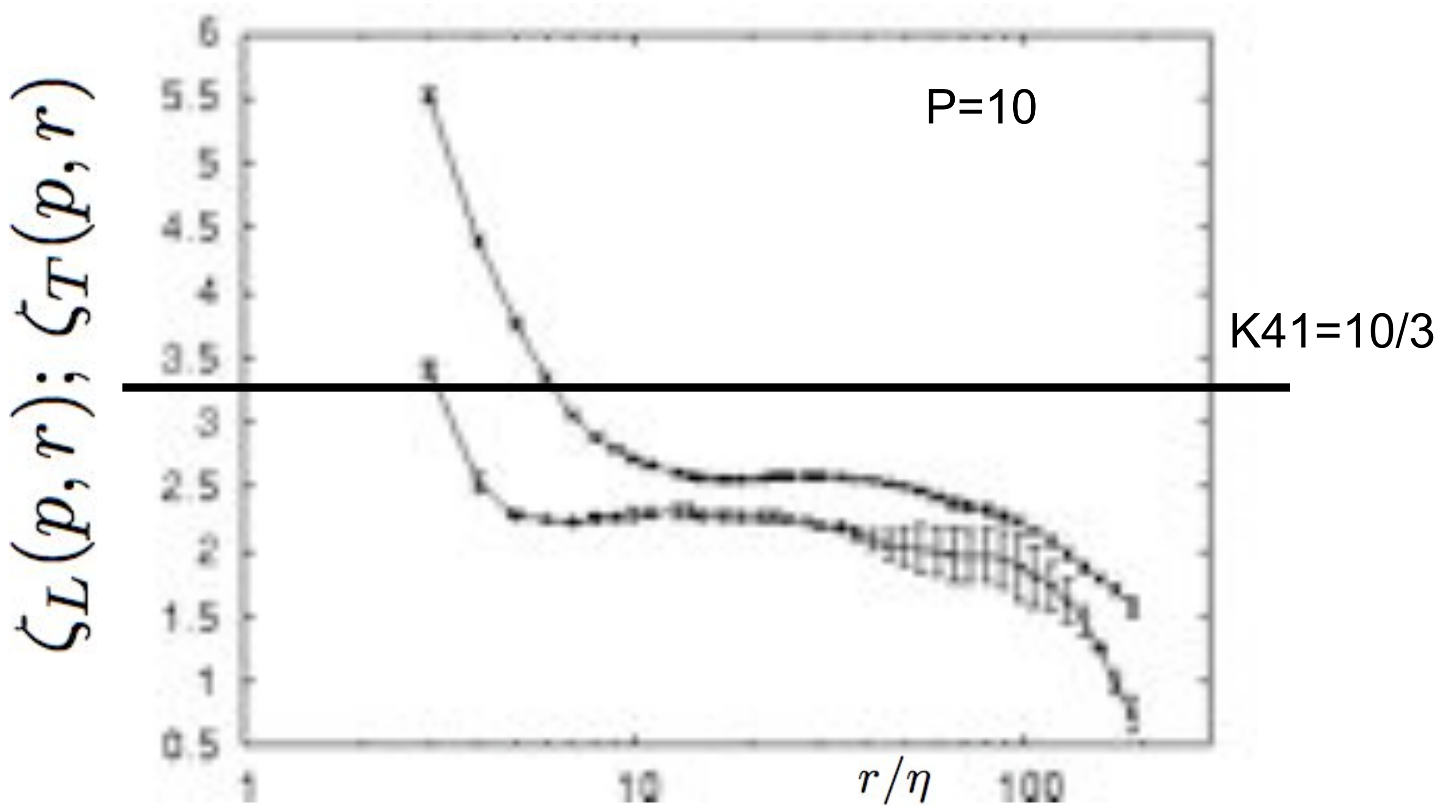
4th: $S_{L,T}^{(2,2)}(r)$ $S_{L,T}^{(0,4)}(r) = S_T^{(4)}(r)$ $S_{L,T}^{(4,0)}(r) = S_L^{(4)}(r)$

EULERIAN STATISTICS: LONGITUDINAL VS TRANSVERSE

$$S_L^{(p)}(r) = \langle (\delta_r u_L)^p \rangle \qquad S_T^{(p)}(r) = \langle (\delta_r u_T)^p \rangle$$

$$\zeta_L(p,r) \overset{\text{DEF}}{=} \frac{d \log \langle (\delta_r u_L)^p \rangle}{d \log r} \qquad \zeta_T(p,r) \overset{\text{DEF}}{=} \frac{d \log \langle (\delta_r u_T)^p \rangle}{d \log r}$$

LOCAL SLOPES: LONGITUDINAL AND TRANSVERSE:

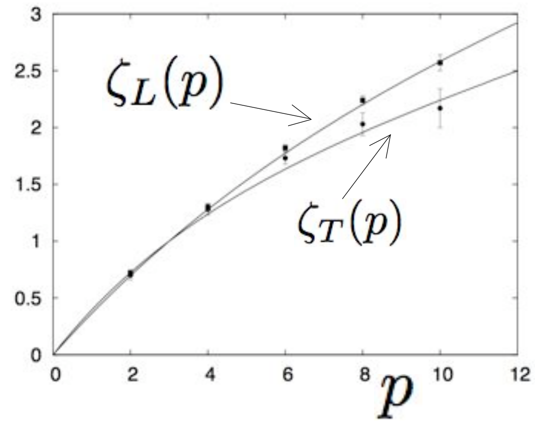


Benzi et al , JFM, 653, p 221 (2010)

LONGITUDINAL AND TRANSVERSE SCALE DIFFERENTLY !

$$\delta_r v \sim v_0 \frac{r}{[(\frac{\eta(h)}{L_0})^\alpha + (\frac{r}{L_0})^\alpha]^{(1-h/)\alpha}} \qquad \mathcal{P}_r(h) = ((\frac{\eta(h)}{L_0})^\alpha + (\frac{r}{L_0})^\alpha)^{\frac{3-D(h)}{\alpha}}$$

$$S_p(r) = \int dh (\delta_r v)^p P_r(h) \qquad \begin{cases} D_L(h) \\ D_T(h) \end{cases}$$



Gotoh et al. (PoF 2002)

Benzi et al , JFM, 653, p 221 (2010)

p	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$ ref. []	$\zeta_T^{(p)}$ ref. []
2	0.71 ± 0.02	0.71 ± 0.02	0.70 ± 0.01	0.71 ± 0.01
4	1.29 ± 0.03	1.27 ± 0.05	1.29 ± 0.03	1.26 ± 0.02
6	1.78 ± 0.04	1.68 ± 0.06	1.77 ± 0.04	1.67 ± 0.04
8	2.18 ± 0.05	1.92 ± 0.10	2.17 ± 0.07	1.93 ± 0.09
10	2.50 ± 0.06	2.10 ± 0.20	2.53 ± 0.09	2.08 ± 0.18

1. Growth of fluctuations by decreasing scale/increasing Reynolds
2. Multi-Step Energy Transfer (cascade)
3. Multiplicative Stochastic Processes
4. Large Deviations Theory <-> Intermittency
5 Multiaffine Fields/Multifractal Measures
6. Fluctuating viscous effects
7. Non-trivial geometrical effects (longitudinal vs transverse)

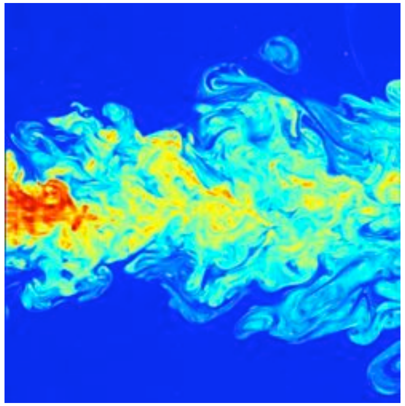
- Toward real world (II): anisotropy.

$$\begin{cases} \partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \boldsymbol{\partial} \boldsymbol{v} = -\boldsymbol{\partial} P + \nu \partial^2 \boldsymbol{v} + \boldsymbol{f} \\ \boldsymbol{\partial} \cdot \boldsymbol{v} = 0 \\ + \text{ boundary conditions} \end{cases}$$

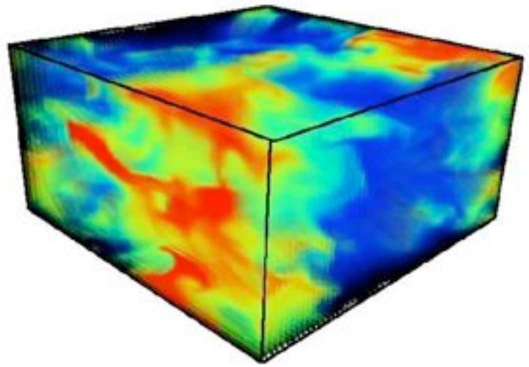
•Kinematics + Dissipation are invariant under Rotation+Translation

•Non-universal statistical behaviour <-> Anisotropy

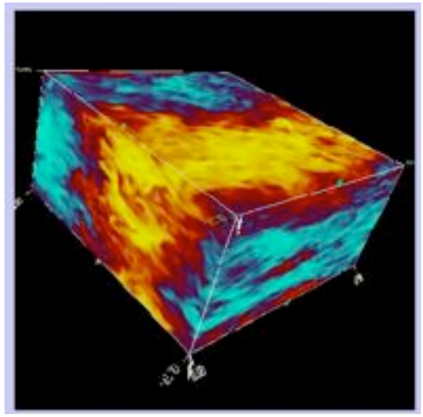
•Small scales vs large scales



Turbulent jet



3d Convective Cell



Shear Flow

I. Arad, V. L'Vov I. Procaccia PRE 59, 6753 (1999).
Arad et al. PRL 82, 5040 (1999)
Arad et al. PRL 81, 5330 (1998).

$$S_n^{\alpha_1\cdots\alpha_n}(\boldsymbol{r}) \stackrel{\text{def}}{=} \langle \delta v^{\alpha_1}(\boldsymbol{x},\boldsymbol{r},t)\cdots\delta v^{\alpha_n}(\boldsymbol{x},\boldsymbol{r},t)\rangle ,$$

$$\delta\boldsymbol{v}(\boldsymbol{x},\boldsymbol{r},t) \stackrel{\text{def}}{=} \boldsymbol{v}(\boldsymbol{x}+\boldsymbol{r},t)-\boldsymbol{v}(\boldsymbol{x},t) ,$$

3d rotation

$$x'_\alpha = \Lambda_{\alpha,\beta} x_\beta$$

Decomposition in terms of (irreducible) invariant subset -labelled by q,j=0,1,2,...

Set of 3n*(2j+1) Eigenfunctions of group of rotations in 3d: $B_{q,jm}^{\alpha_1\ldots\alpha_n}(\mathbf{r})$

n-rank tensor which depends on a 3d vector

$$S_n^{\alpha_1\cdots\alpha_n}(\boldsymbol{r}) = \quad = \sum_{qjm} \mathcal{S}_{qjm}(r) B_{qjm}^{\alpha_1\cdots\alpha_n}(\hat{\boldsymbol{r}}) .$$

The simplest set of 0-rank tensor (SCALAR) observable:

Longitudinal Structure Functions

$$S^{(n)}(\boldsymbol{r}) = \langle [(\boldsymbol{v}(\mathrm{x}+\boldsymbol{r}) - \boldsymbol{v}(\mathrm{x})) \cdot \hat{\boldsymbol{r}}]^n \rangle \,.$$

$$S^{(n)}(\boldsymbol{r}) = \sum_{j=0}^{\infty} \sum_{m=-j}^{j} \mathcal{S}_{jm}^{(n)}(r) Y_{jm}(\hat{\boldsymbol{r}}).$$

FIGURE 4. Graphical representation of spherical harmonics (a) $|Y^{2,0}(\theta,\phi)|$, (b) $|Y^{2,1}(\theta,\phi)|$, and (c) $|Y^{2,2}(\theta,\phi)|$.

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \boldsymbol{\partial v} = -\boldsymbol{\partial} P + \nu \partial^2 \boldsymbol{v} + \boldsymbol{f}$$

$$\partial_t v_i + \Gamma_{ijk}(v_j v_k) - \nu \Delta v_i = f_i$$

$$\partial_t S^n + \Gamma^{n+1} S^{n+1} - \nu D^n S^n = \langle \delta f_1 \delta v_2 \cdots \delta v_{n-1} \rangle + perm.$$

rotational invariant operator

$r << L_f$

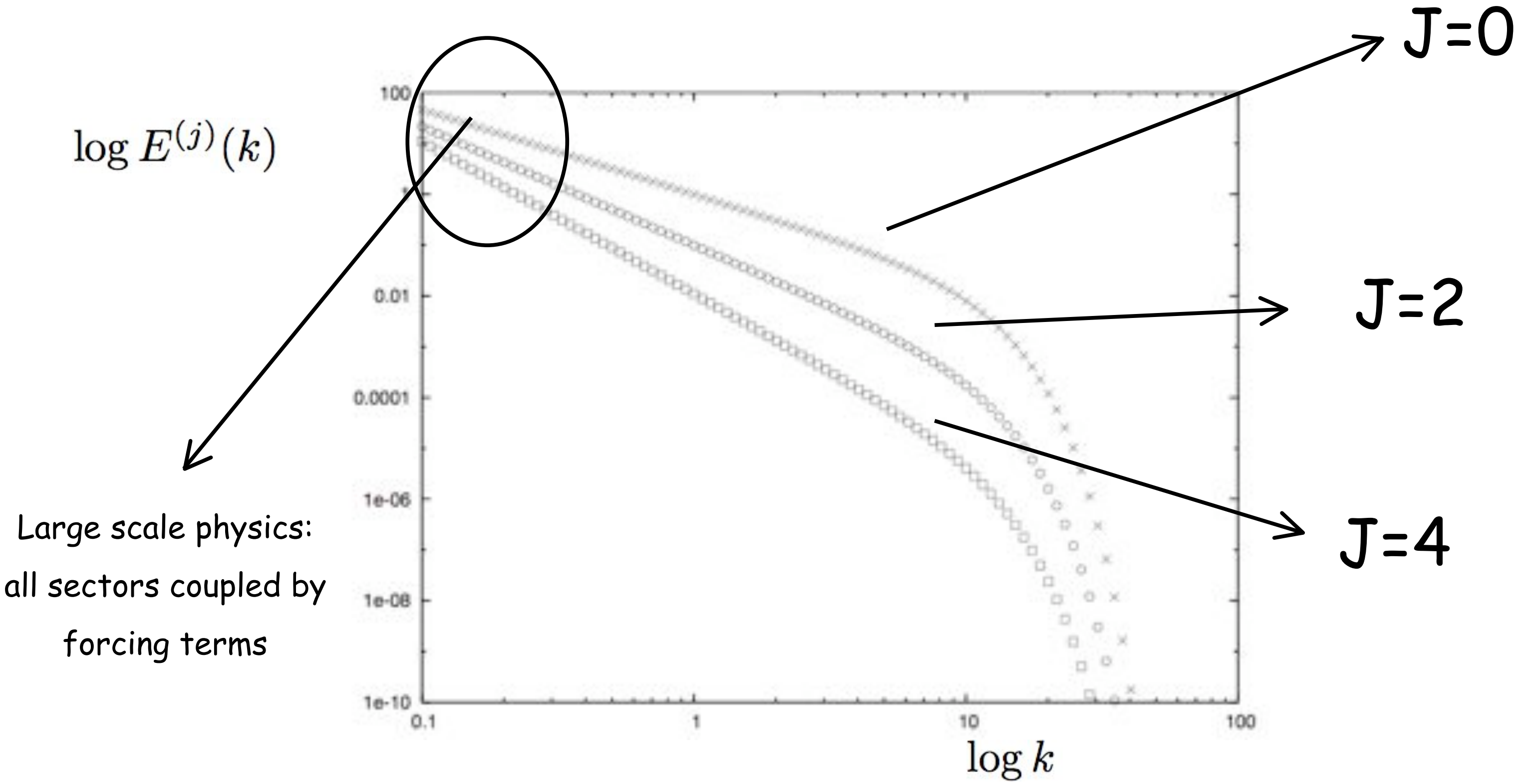
$$\partial_t S^n + \Gamma^{n+1} S^{n+1} - \nu D^n S^n \sim 0$$

+ so(3) -> $S^{(n)}_{\alpha_1 \ldots \alpha_n}(\mathbf{r}) = \sum_{jmq} \mathcal{S}^{(n)}_{jmq}(r) B^{jmq}_{\alpha_1 \ldots \alpha_n}(\hat{r})$

$$\partial_t \mathcal{S}^n_{jmq} + \sum_{q'} \Gamma^{n+1}_{jmq'} \mathcal{S}^{n+1}_{jmq'} - \nu D^n_{jmq} \mathcal{S}^n_{jmq} = 0$$

FOLIATION !!!

$$\partial_t \mathcal{S}^{(n)}_{jq} + \sum_{q'} \Gamma^{(n+1)}_{jq'} \mathcal{S}^{(n+1)}_{jq'} - \nu D^{(n)}_{jq} \mathcal{S}^{(n)}_{jq} \sim 0$$



$$\nu \to 0 \qquad \partial_t \mathcal{S}^{(n)}_{jq} + \sum_{q'} \Gamma^{(n+1)}_{jq'} \mathcal{S}^{(n+1)}_{jq'} - \nu D^{(n)}_{jq} \mathcal{S}^{(n)}_{jq} \sim 0$$

$$\mathcal{S}^{(n)}_{jmq}(r) \propto a_{jmq} \left(\frac{r}{L}\right)^{\zeta^j_n}$$ scaling ?

$$\mathcal{S}_{jqm}^{(n)}(r) \sim A_{jmq}(\frac{r}{L})^{\xi_n^j}$$

Working Hypothesis

•projection on each sector has a **universal** scaling exponent, depending on that sector **only**.

•Dependency on large scale physics shows up only in **prefactors**

•Pure power laws **only** in each separeted sector:

$$S^{(n)}(\mathbf{r}) \sim \sum_j A_j(\frac{r}{L})^{\zeta_n^j} \longrightarrow S^{(n)}(\mathbf{r}) \sim A_0(\frac{r}{L})^{\zeta_n^0} + A_1(\frac{r}{L})^{\zeta_n^1} + \cdots$$

$$S^{(n)}(\mathbf{r}) \sim A_0(\frac{r}{L})^{\zeta_n^0} + A_1(\frac{r}{L})^{\zeta_n^1} + \cdots$$

•Matching Infra-Red boundary conditions: $r \sim L$

$$S^{(n)}(\mathbf{L}) \sim A_0 + A_1 + A_2 + \cdots$$

prefactor <u>cannot be universal</u>

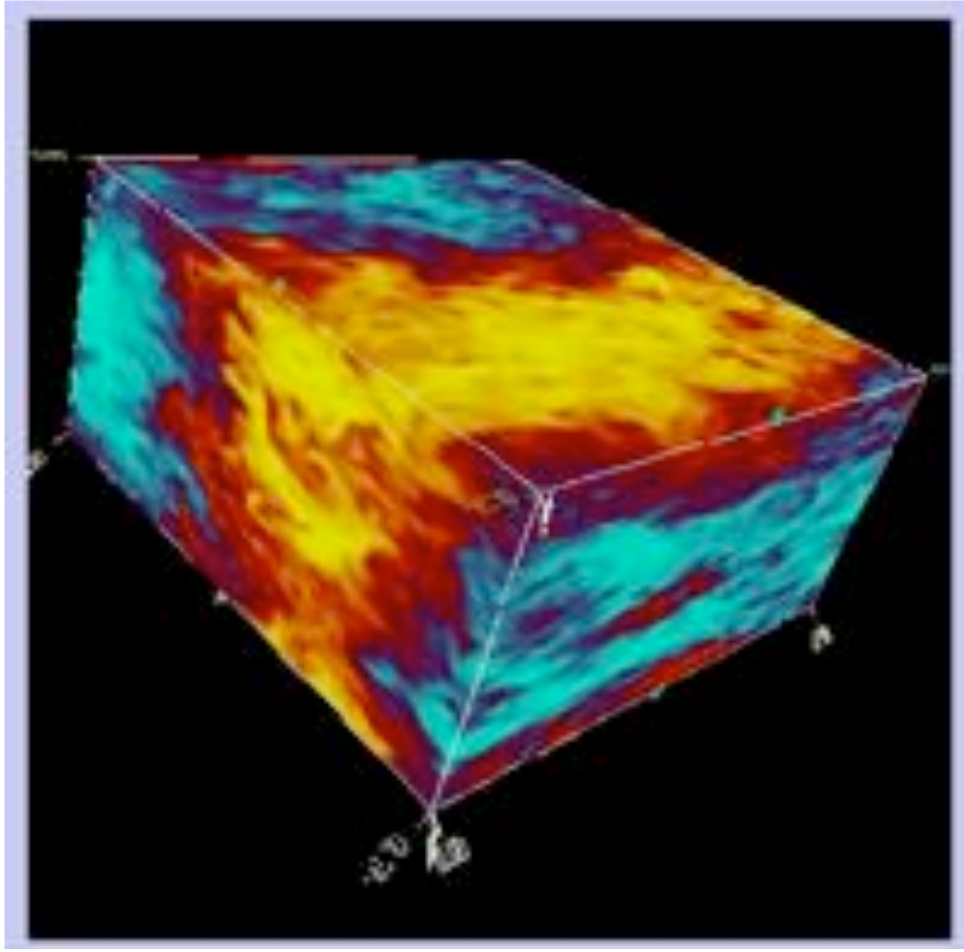
•About universality of scaling exponents nothing can be said rigorously, at least for the NS eqs.

•Recovery of Isotropy

•Small-Scales Universality

$$\zeta^{j=0}(n) \leq \zeta^{j=1}(n) \leq \zeta^{j=2}(n) < \ldots$$

Scaling in anisotropic sectors



L.B. and F. Toschi, PRL 86, 4831 (2001)

L.B. I. Daumont, A. Lanotte and F. Toschi. PRE. 66, 056306 (2002)

We performed a DNS
of a Random-Kolmogorov Flow

•Periodic boundary conditions

•256x256x256

•Hyperviscosity

•Homogeneous but
Anisotropic

$$f_z = cos(z + \phi(t))$$

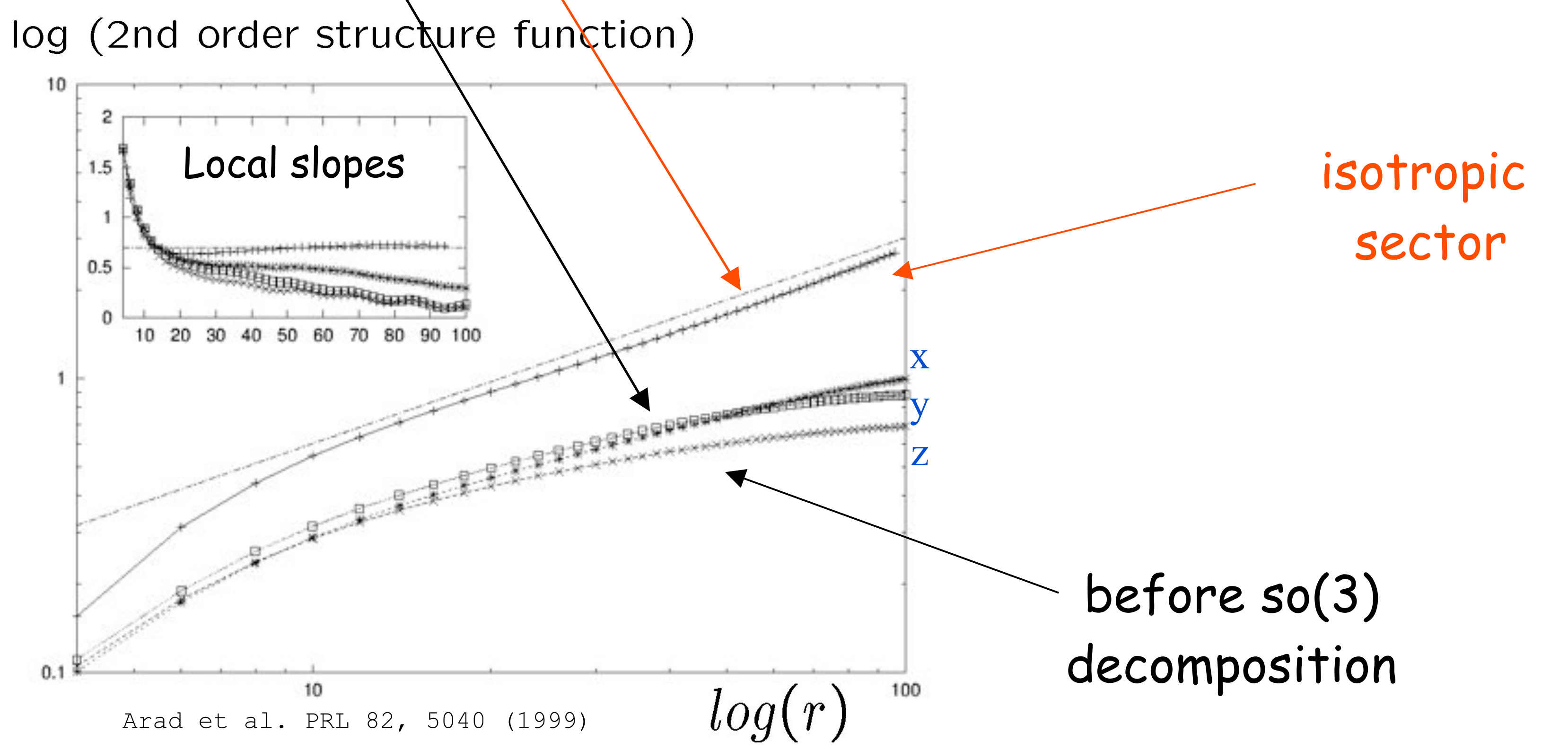
$$\langle \phi(t)\phi(t')\rangle = \delta(t - t')$$

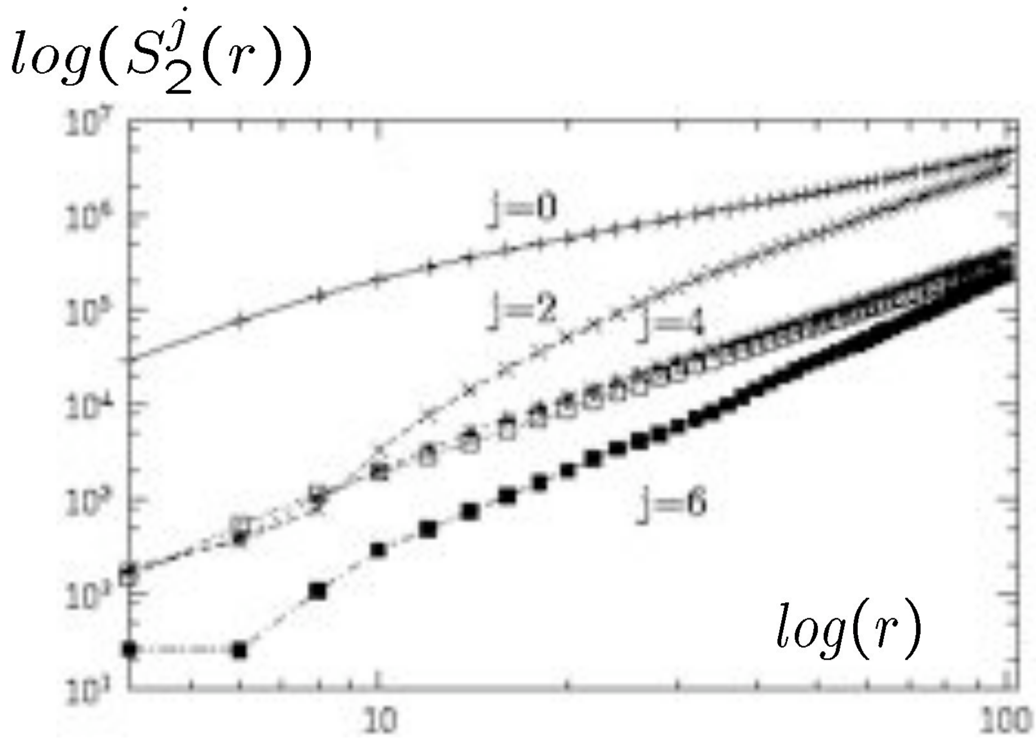
Comparison of scaling properties:

isotropic sector (j=0,m=0) vs undecomposed structure function

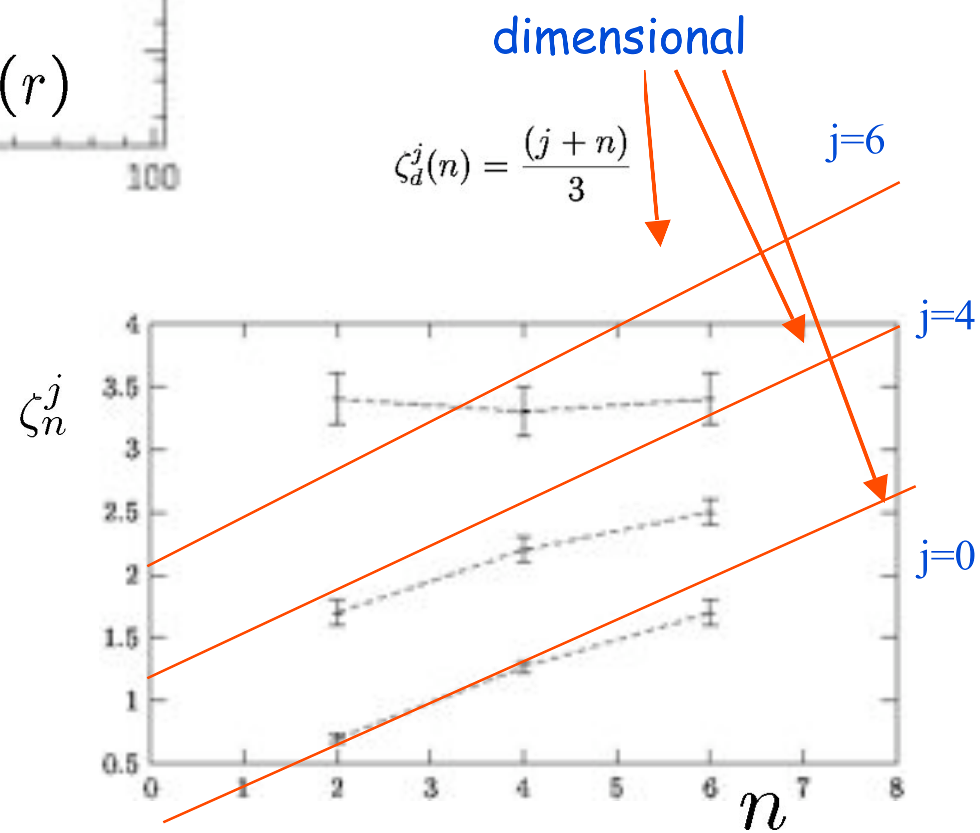
$$S_n(\mathbf{r}) = \sum_{j=0}^{\infty} \sum_{m=-j}^{j} S_n^{jm}(|r|) Y_{jm}(\hat{r})$$

$$S_n(\mathbf{r}) = a_0 r^{\zeta_n^{j=0}} + a_2 r^{\zeta_n^{j=2}} + a_4 r^{\zeta_n^{j=4}} + \cdots$$





scaling is m-independent



$$\zeta_n^{j=0} < \zeta_n^{j=1} < \zeta_n^{j=2} < \cdots$$

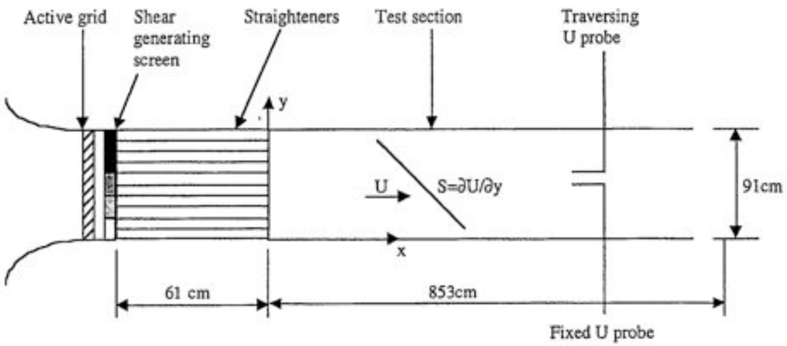
Recovery of isotropy

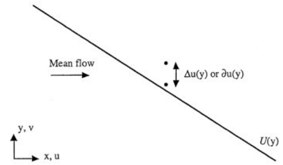
L.B. I. Daumont, A> Lanotte and F. Toschi
PRE. 66, 056306 (2002)

Recovery of isotropy vs persistency of Anisotropies

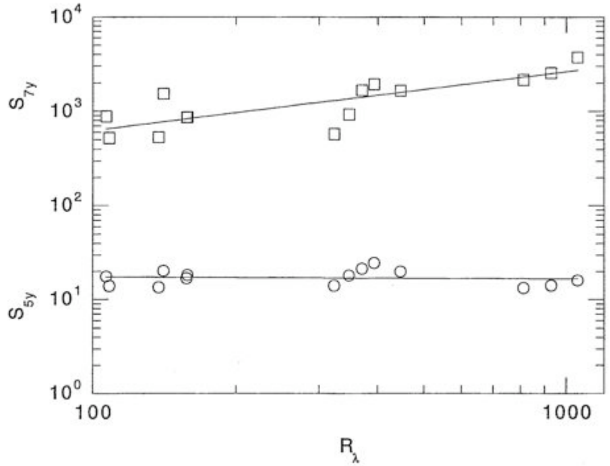
Experimental Results on Persistency of Anisotropies

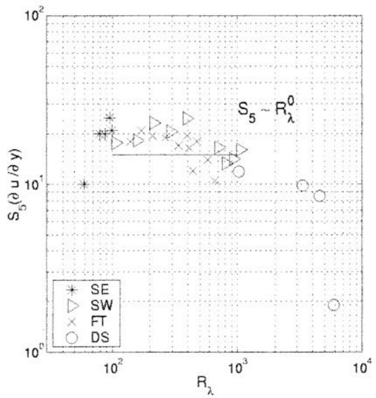
Garg and Warhaft, PoF 10, 662 (1998).
Kurien et al. PRE 61, 407 (2000).
Kurien and Sreenivasan, PRE 62, 2206 (2000).
Shen and Warhaft, PoF 14, 370 and 2432 (2002).



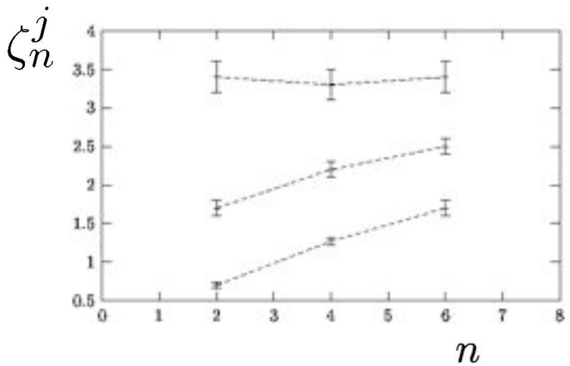


$$S^{(2n+1)}(R_\lambda) = \frac{\langle(\partial_y v_x)^{2n+1}\rangle}{\langle(\partial_y v_x)^2\rangle^{\frac{2n+1}{2}}}. = \frac{\text{anis.}}{\text{iso}}$$





$$S^{(n)}(r,\hat{r}) = A_0 r^{\zeta_n^{j=0}} + A_2(\hat{r}) r^{\zeta_n^{j=2}} + A_4(\hat{r}) r^{\zeta_n^{j=4}} + \cdots$$



$$\zeta_n^{j=0} \leq \zeta_n^{j=1} \leq \zeta_n^{j=2} < \dots$$

Two ways to measure small-scales anisotropies:

aniso(n)/iso(n)

1)

$$F_j^{(n)}(r) = \frac{\mathcal{S}_j^{(n)}(r)}{\mathcal{S}_{j=0}^{(n)}(r)} \sim r^{\Delta_n^j} \to 0; \quad \Delta_n^j = \zeta_n^j - \zeta_n^0 > 0$$

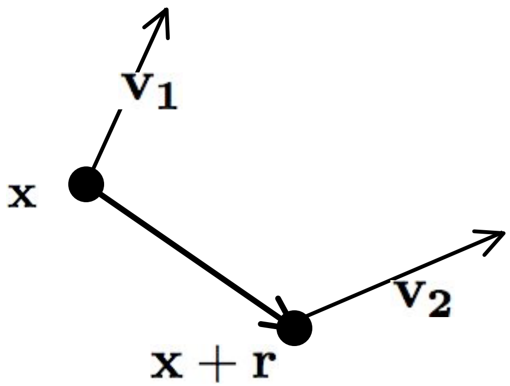
aniso(n)/iso(n=2)

2)

$$K_j^{(n)}(r) = \frac{\mathcal{S}_j^{(n)}(r)}{(\mathcal{S}_{j=0}^{(2)}(r))^{\frac{n}{2}}} \sim r^{\Delta_n^j} \to ?; \quad \Delta_j^n = \zeta_n^j - \frac{n}{2}\zeta_2^0 <=> 0$$

L.B. and M. Vergassola, PoF. 13, 2139 (2001)

Open questions



$$\delta_L v(r) = (\mathbf{v_1} - \mathbf{v_2}) \cdot \mathbf{r}$$
$$\delta_T v(r) = (\mathbf{v_1^\perp} - \mathbf{v_2^\perp})$$

<u>fully isotropic</u>

n=2
J=0

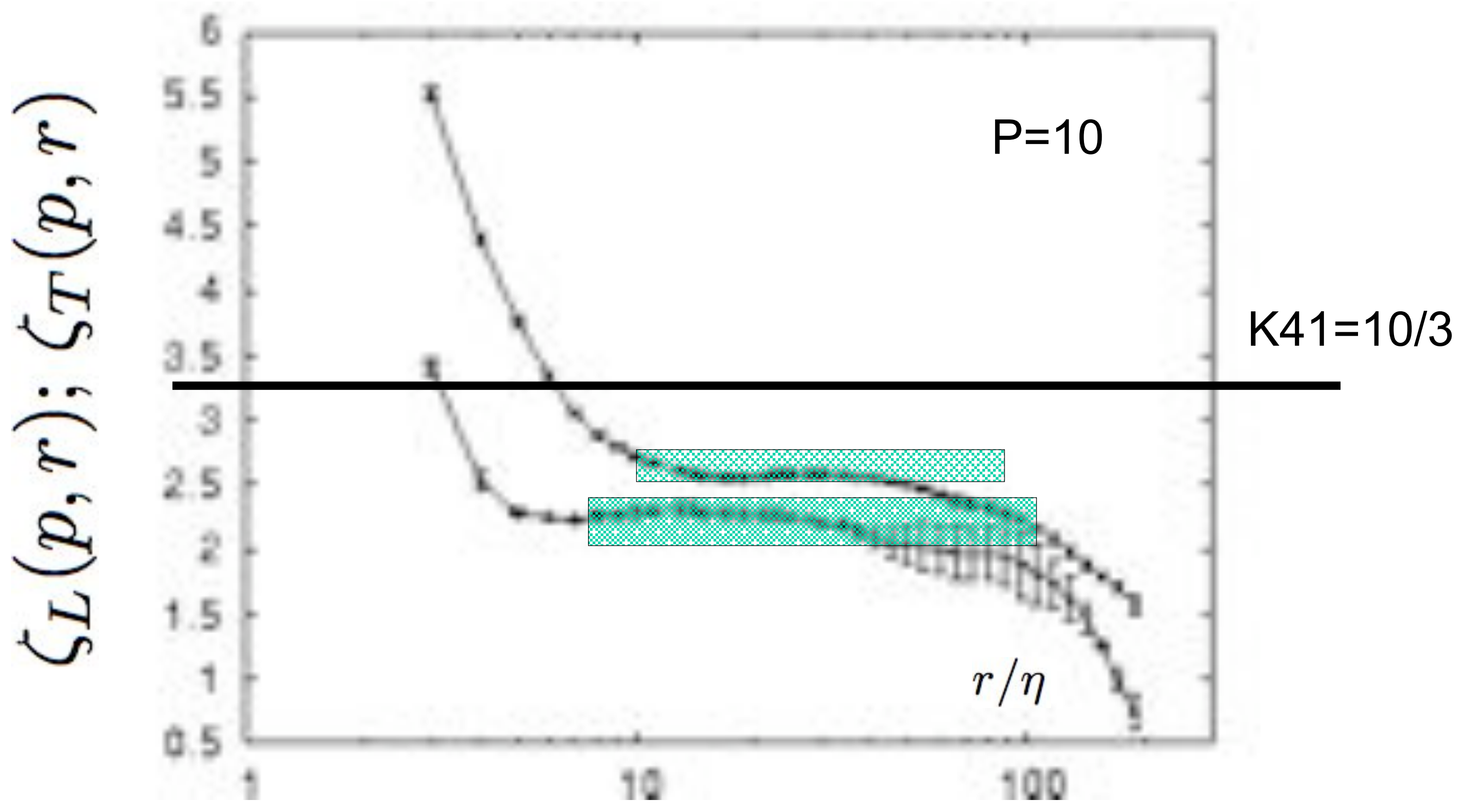
$$\begin{cases} \langle \delta v^\alpha(\mathbf{r}) \delta v^\beta(\mathbf{r}) \rangle = a(r)\hat{r}^\alpha \hat{r}^\beta + b(r)\delta^{\alpha\beta} \\ S_L^{(2)}(r) = \langle (\delta_L v)^2 \rangle = a(r) + b(r) \\ S_T^{(2)}(r) = \langle (\delta_T v)^2 \rangle = b(r) \end{cases}$$

n=4
J=0

$$\begin{cases} \langle \delta v^\alpha \delta v^\beta \delta v^\gamma \delta v^\delta \rangle \sim c(r)\hat{r}^\alpha \hat{r}^\beta \hat{r}^\gamma \hat{r}^\delta + d(r)[\hat{r}^\alpha \hat{r}^\beta \delta^{\gamma\delta} + perm] + e(r)[\delta^{\gamma\delta}\delta^{\alpha\beta} + perm] \\ S_L^{(4)}(r) = \langle (\delta_L v)^4 \rangle = c(r) + 3d(r) + 3e(r) \\ S_T^{(4)}(r) = \langle (\delta_T v)^4 \rangle = 3e(r) \\ S_{LT}^{(4)}(r) = \langle (\delta_T v)^2 (\delta_L v)^2 \rangle = 3d(r) + 3e(r) \end{cases}$$

Benzi et al , JFM, 653, p 221 (2010)

1st MESSAGE: LONGITUDINAL AND TRANSVERSE SCALE DIFFERENTLY



Gotoh et al. (PoF 2002)

p	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$ ref. []	$\zeta_T^{(p)}$ ref. []
2	0.71 ± 0.02	0.71 ± 0.02	0.70 ± 0.01	0.71 ± 0.01
4	1.29 ± 0.03	1.27 ± 0.05	1.29 ± 0.03	1.26 ± 0.02
6	1.78 ± 0.04	1.68 ± 0.06	1.77 ± 0.04	1.67 ± 0.04
8	2.18 ± 0.05	1.92 ± 0.10	2.17 ± 0.07	1.93 ± 0.09
10	2.50 ± 0.06	2.10 ± 0.20	2.53 ± 0.09	2.08 ± 0.18

•SO(3) decomposition is needed if you want to disentangle in a systematic way isotropic from anisotropic contributions and different anisotropic contributions among themselves.

•Dynamical importance through the "foliation" mechanism of the eqs. of motion.

•(i) Power law behaviour only in separated (j) sectors; (ii) intermittency also in anisotropic sectors, (iii) (slow) Recovery of small-scales isotropy.

•OPEN QUESTIONS: (i) Universality of anisotropic exponents? (ii) longitudinal vs transverse scaling in isotropic sector.

For a recent review see:

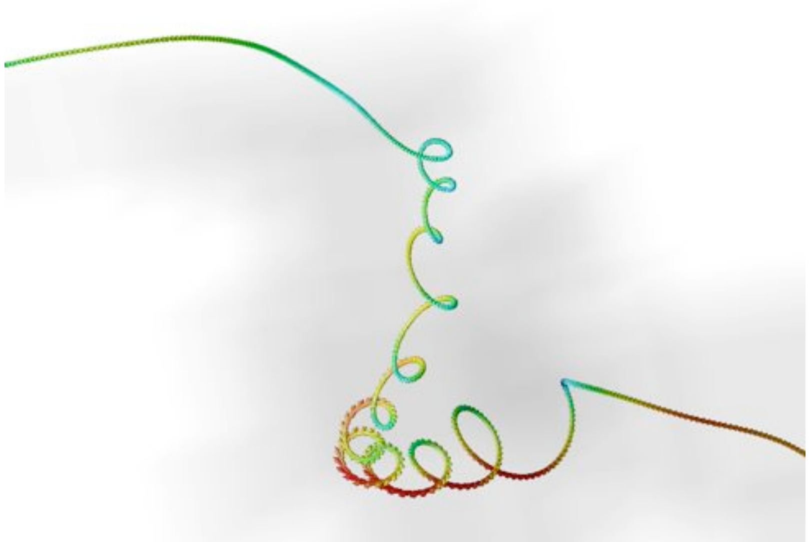
L. Biferale & I. Procaccia

Anisotropy in turbulent flows and in turbulent transport

Physics Reports Volume 414, Issues 2-3 , July 2005, Pages 43-164

•Toward real world (III): Turbulence + passive particles (tracers, heavy, light).

Lagrangian



Lagrangian turbulence?

Is the multifractal/ldt formalism able to describe also the phenomenology of Lagrangian turbulence ?

"….Unfortunately, there are no significant lagrangian measurements of velocity, acceleration, etc., to test the multifractal predictions. …"

M.S. Borgas, "The Multifractal Lagrangian Nature of Turbulence", Phyl. Trans: Phys. Sciences and Eng. Vol. 342 (**1993**) 379.

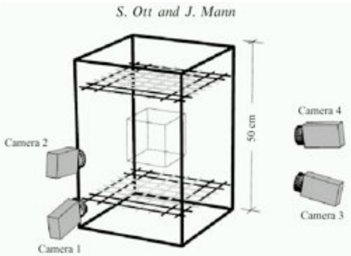
Recently things are changing !

Eulerian MF   Lagrangian MF

With some surprise…

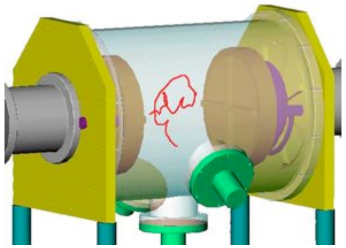
Experiments

Experimental Lagrangian measurements are intrinsically difficult: one has to follow (many) Lagrangian trajectories for long time at high Reynolds (i.e. high sampling frequency)

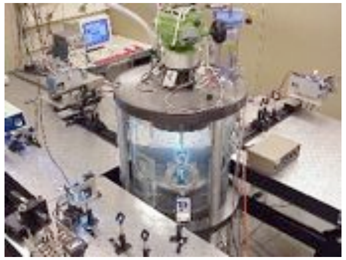


Ott and Mann experiment at Risø
conventional 3D PTV - Re_λ=100-300

Luthi, Tsinober et al
3D PTV and 3D scanning PTV for velocity gradients



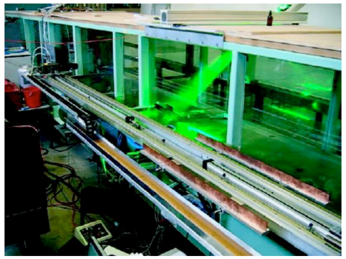
Pinton et al ENSL
Acoustic/Laser Doppler tracking - Re_λ ~800 (single particle tracking)



Bodenschatz et al at Cornell-MPI
silicon strip detectors (now also CCD) $Re_\lambda \approx 1000$-1500

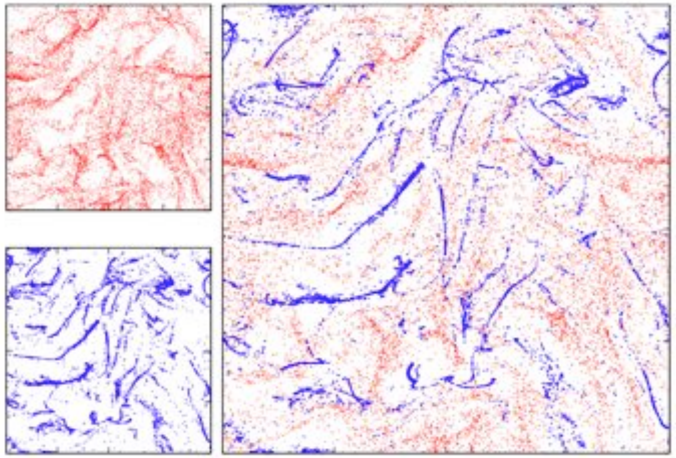
non intrusive tracking down to

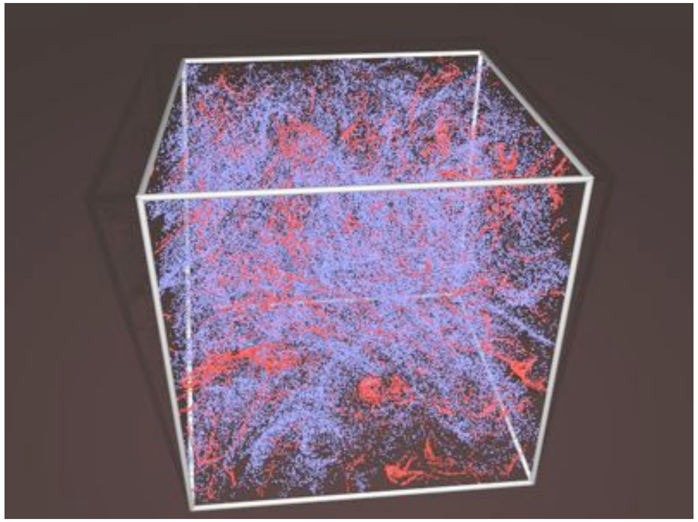
$$\tau \sim \tau_\eta$$



Warhaft et al experiment at Cornell
Fast moving camera
$Re_\lambda \approx 300$

DNS





+s and -s

- low to moderate Reynolds numbers, Re
- computationally expensive (Cpu time $\propto Re_\lambda^6$)
- memory demanding (ram $\propto Re_\lambda^{9/2}$)

+ high time resolution and long tracking $\quad \tau \ll \tau_\eta$
+ large Lagrangian statistics
+ multiparticle tracking
+ simultaneous Eulerian-Lagrangian statistics

Lagrangian velocity statistics

$$S_p(r) = \langle (u(x+r) - u(x))^p \rangle \sim r^{\zeta_E(p)}$$

$$S_p(\tau) = \langle (v(t+\tau) - v(t))^p \rangle \sim \tau^{\zeta_L(p)}$$

$$\tau_\eta \ll \tau \ll T_L$$

Does it exist and how to estimate $\zeta_L(p)$?

In Eulerian turbulence we have $\zeta_E(p) = \inf_h (hp + 3 - D(h))$

Let's try to make a **predictions**

$$\delta_\tau v \sim \delta_r u$$

We assume that r and τ are linked by the typical eddy turn over time at the given spatial scale

$$\tau_r \sim r/\delta_r u$$

Bridge between Eulerian and Lagrangian description:

$$\tau \sim \frac{L_0^h}{v_0} r^{1-h}$$

[Borgas (1993); Boffetta et al (2002)]

$$S_p(r) = \langle (\delta_r v)^p \rangle \sim \langle v_0^p \rangle \int_I dh \left(\frac{r}{L_0}\right)^{hp+3-D(h)}$$ EULERIAN

Multifractal prediction for the Lagrangian structure functions

$$S_p(\tau) \sim \langle v_0^p \rangle \int_{h\in I} \mathrm{d}h \left(\frac{\tau}{T_L}\right)^{\frac{hp+3-D(h)}{1-h}}$$

where

$$\zeta_L(p) = \inf_h \left(\frac{hp+3-D(h)}{1-h}\right)$$

Same D(h) of the Eulerian field !!

WARNING: NO EXACT RESULTS SUPPORTING THE EXISTENCE OF SCALING LAWS IN LAGRANGIAN FRAMEWORK

BATCHELOR-MENEVEAU -> LAGRANGIAN

[CHEVILLARD ET AL PRL 2003]

$$\delta_\tau v = v_0 \frac{\tau/T_L}{[(\frac{\tau}{T_L})^\beta + (\frac{\tau_\eta}{T_L})^\beta]^{\frac{1-2h}{\beta(1-h)}}}$$

$$\delta_\tau v \sim \left(\frac{\tau}{T_L}\right)^{\frac{h}{1-h}}$$

$$\tau \gg \tau_\eta$$

$$\delta_\tau v \sim \tau \frac{\delta_{\tau_\eta} v}{\tau_\eta} \sim a\tau$$

$$\tau \ll \tau_\eta$$

β free parameter

$$\mathcal{P}_h(\tau, \tau_\eta) \sim \left[(\frac{\tau_\eta}{T_L})^\beta + (\frac{\tau}{T_L})^\beta\right]^{\frac{3-D(h)}{\beta(1-h)}}$$

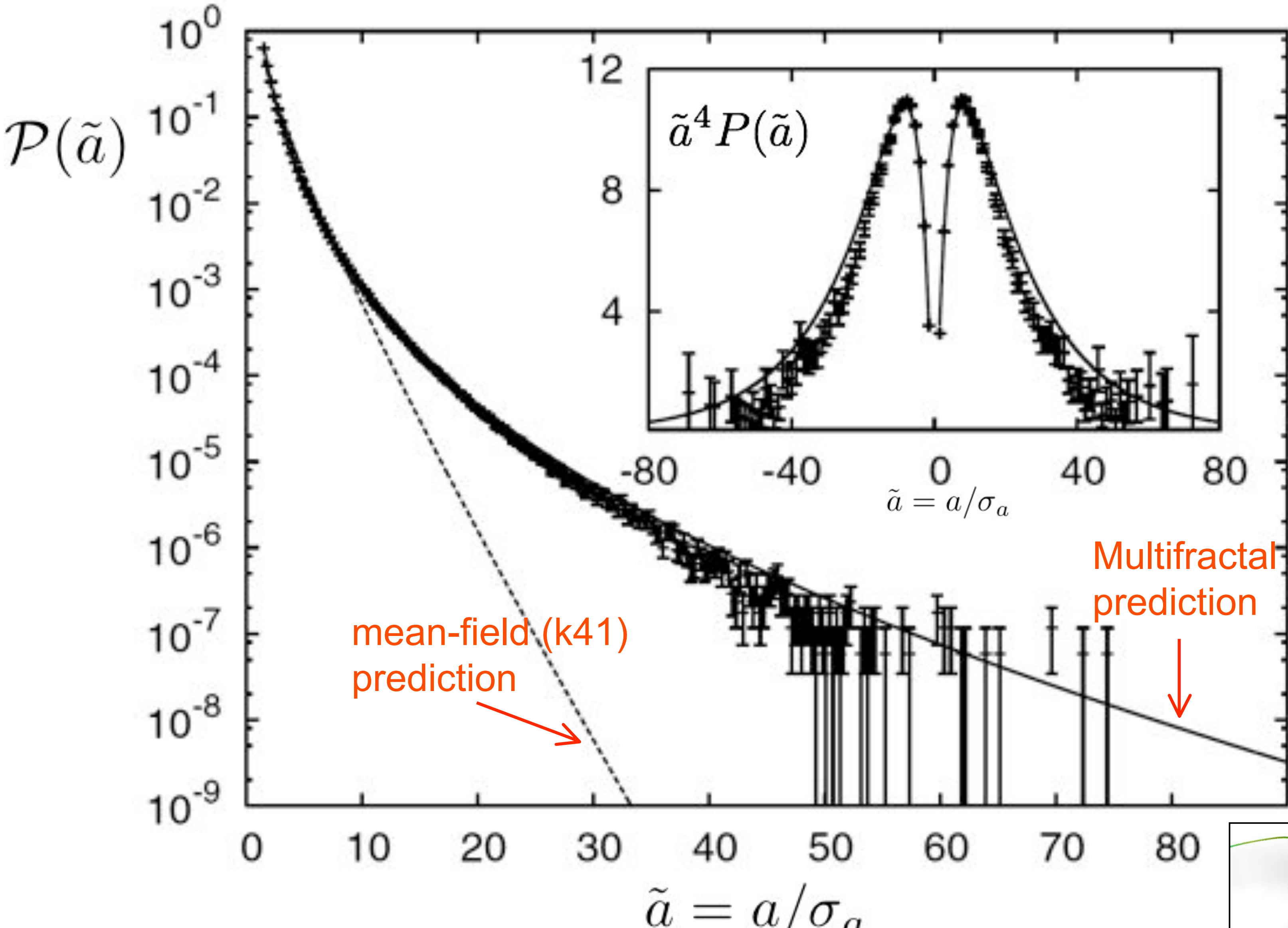
Start from Eulerian $D(h)$

but: dissipative time fluctuates (as the dissipative scale)

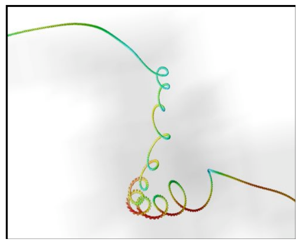
$$\tau_\eta(h) \sim Re_\lambda^{\frac{2(h-1)}{1+h}}$$

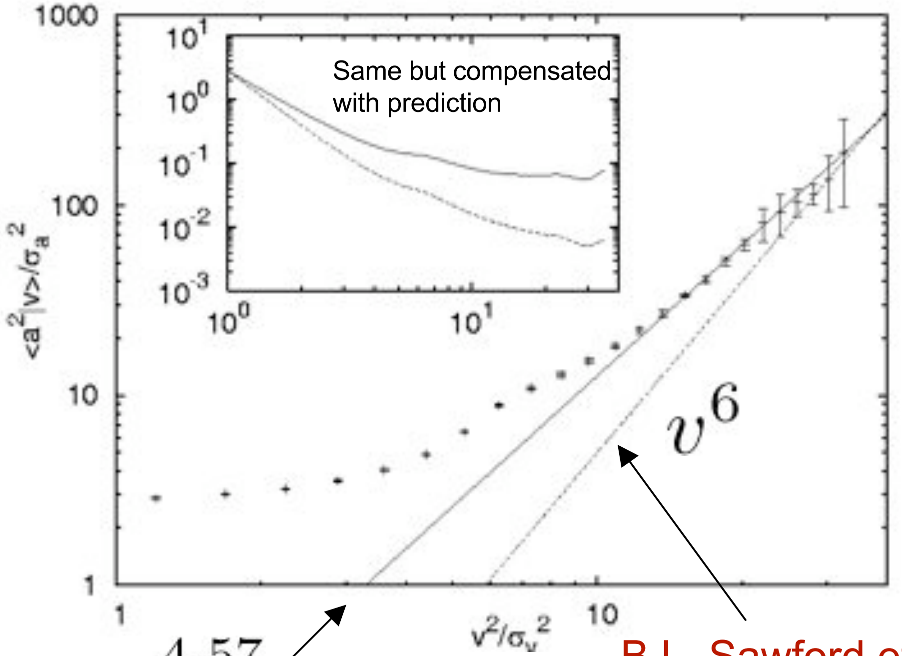
[L.B., G. Boffetta, A. Celani, B. Devenish, A.S. Lanotte & F. Toschi PRL 2004

Acceleration PDF



$$\mathcal{P}(a) \sim \int_{h \in I} \mathrm{d}h\, a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \quad \exp\left(- \frac{a^{\frac{2(1+h)}{3}} \nu^{\frac{2(1-2h)}{3}} L_0^{2h}}{2\sigma_v^2} \right)$$





$v^{4.57}$

Multifractal prediction

B.L. Sawford et al., Phys. Fluids **15**, 3478 (2003).

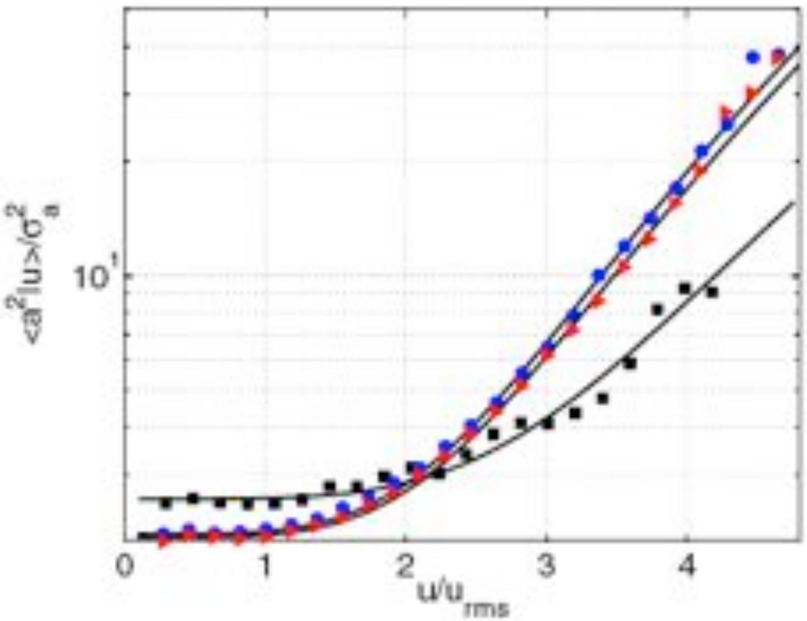
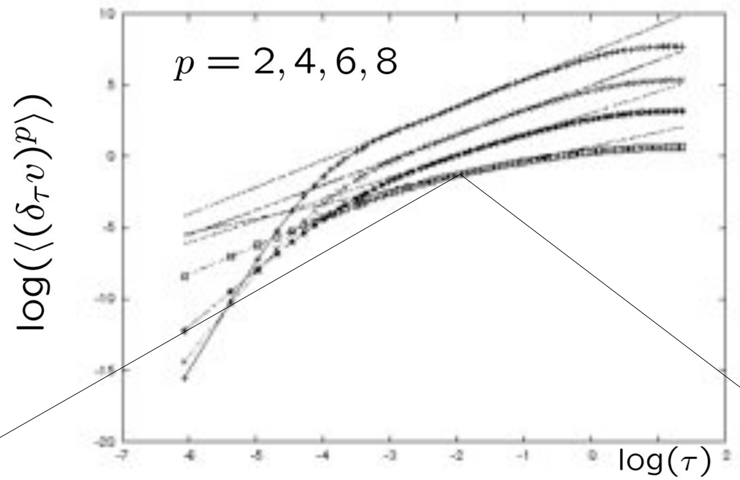


FIG. 4 (color online). Normalized conditional acceleration variance $\langle a^2|u\rangle/\sigma_a^2$ for $R_\lambda = 690$, 485, 285, circles, triangles, and squares, respectively. Solid lines are the fit (3).

Joint Statistics of the Lagrangian Acceleration and Velocity in Fully Developed Turbulence. Crawford, Mordant, and Bodenschatz PRL 94, 024501 (2005)



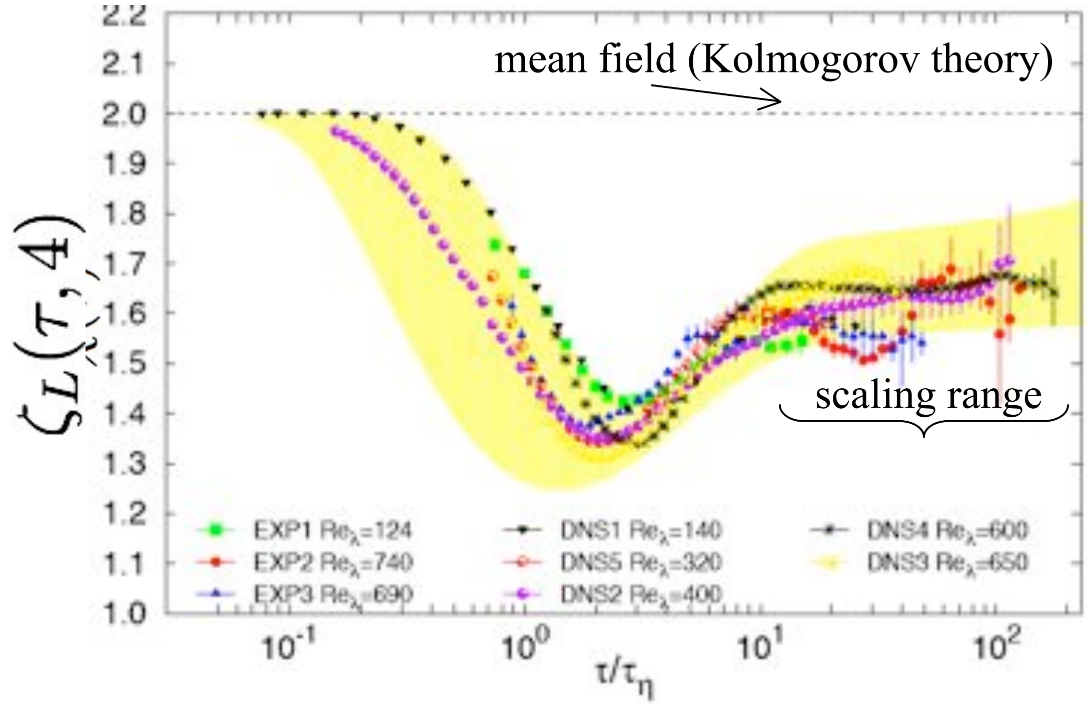
$$\zeta_p(\tau) = \frac{\mathrm{d}\log\left(S_p(\tau)\right)}{\mathrm{d}\log\left(S_2(\tau)\right)}$$

The local exponents $\zeta_p(\tau)$ act as **magnifying glass**, probing locally the value of the scaling exponents.

-) Power law scaling -> plateaux for **local scaling exponents**

-) Comparing results from different components: estimate of anisotropy

$$F(\tau) = \langle(\delta_\tau v)^2\rangle^{(\zeta_L(\tau,4)-2)}$$



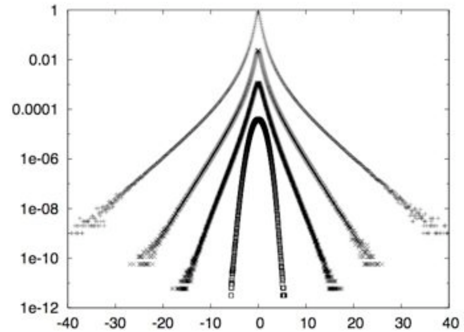


FIG. 1: Log-Lin plot of the local exponent for the fourth moment, $\zeta(4,\tau)$, averaged over the three velocity components, as a function of the normalised time lag τ/τ_η. Data sets come from three experiments (EXP) (see table 1) and five direct numerical simulations (DNS) (see table 2). Error bars are estimated out of the spread between the three components, but for EXP1 and EXP3 where only two components have been considered because of large systematic anisotropic effects in the third one. Each data set is plotted only in the time range where the known experimental/numerical limitations are certainly not affecting the results. In particular, for each data set, the largest time lag always satisfies $\tau < T_L$. The minimal time lag is set by the highest fully resolved frequency. The shaded area displays the prediction obtained by the MF model by using $D_L(h)$ or $D_T(h)$, with $\beta = 4$, for a range of $Re_\lambda \in [150:800]$, comparable with the range of Reynolds in the data. Notice that the MF predictions have been obtained by fixing equal to 7, the multiplicative constant in the definition of τ_η (see Methods). The straight dashed line corresponds to the dimensional non-intermittent value $\zeta(4,\tau) = 2$. Notice that two DNS are even resolved enough to get the right dimensional scaling in the high frequency limits.

International Collaboration for Turbulence Research, A. Arneodo,[1] J. Berg,[2] R. Benzi,[3] L. Biferale,[3] E. Bodenschatz,[4] A. Busse,[5] E. Calzavarini,[6] B. Castaing,[1] M. Cencini,[7] L. Chevillard,[1] R. Fisher,[8] R. Grauer,[9] H. Homann,[9] D. Lamb,[8] A.S. Lanotte,[10] E. Leveque,[1] B. Lüthi,[11] J. Mann,[2] N. Mordant,[12] W.-C. Müller,[5] S. Ott,[2] N. Oullette,[13] J.-F. Pinton,[1] S.B. Pope,[14] S.G. Roux,[1] F. Toschi,[15,16] H. Xu,[4] and P.K. Yeung[17]

[PRL 100, 254504 2008]

INFINITELY-MANY ANOMALOUS SCALING EXPONENTS

(MULTIFRACTAL FIELD, Parisi & Frisch, 1983)

$$S_p(\tau) = \langle (v(t+\tau) - v(t))^p \rangle \sim \tau^{\zeta_L(p)}$$

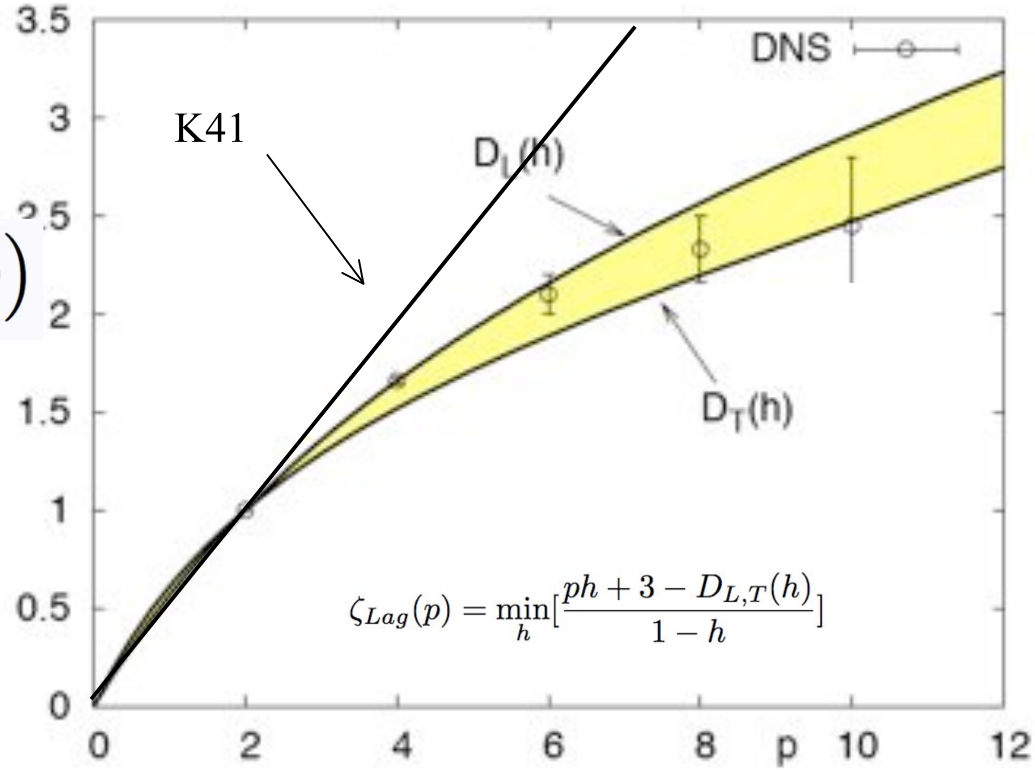
in the scaling range:

$$\zeta_L(\tau, p) \rightarrow \zeta_L(p)$$

$$\begin{cases} t \rightarrow \lambda^{1-h}\, t \\ v \rightarrow \lambda^{h} v \qquad \forall h \\ r \rightarrow \lambda r \end{cases}$$

$D(h)$ multifractal specrtum of local scaling exponents

$\zeta_L(p)$



R. Benzi, L.B. R. Fischer, L. Kadanoff, D. Lamb, F. Toschi PRL 100, 234503 2008.

R. Benzi, L. B., R. Fisher D. Lamb and F. Toschi, JFM 653, p. 221 (2010).

WHAT HAPPENS AROUND DISSIPATIVE TIME?

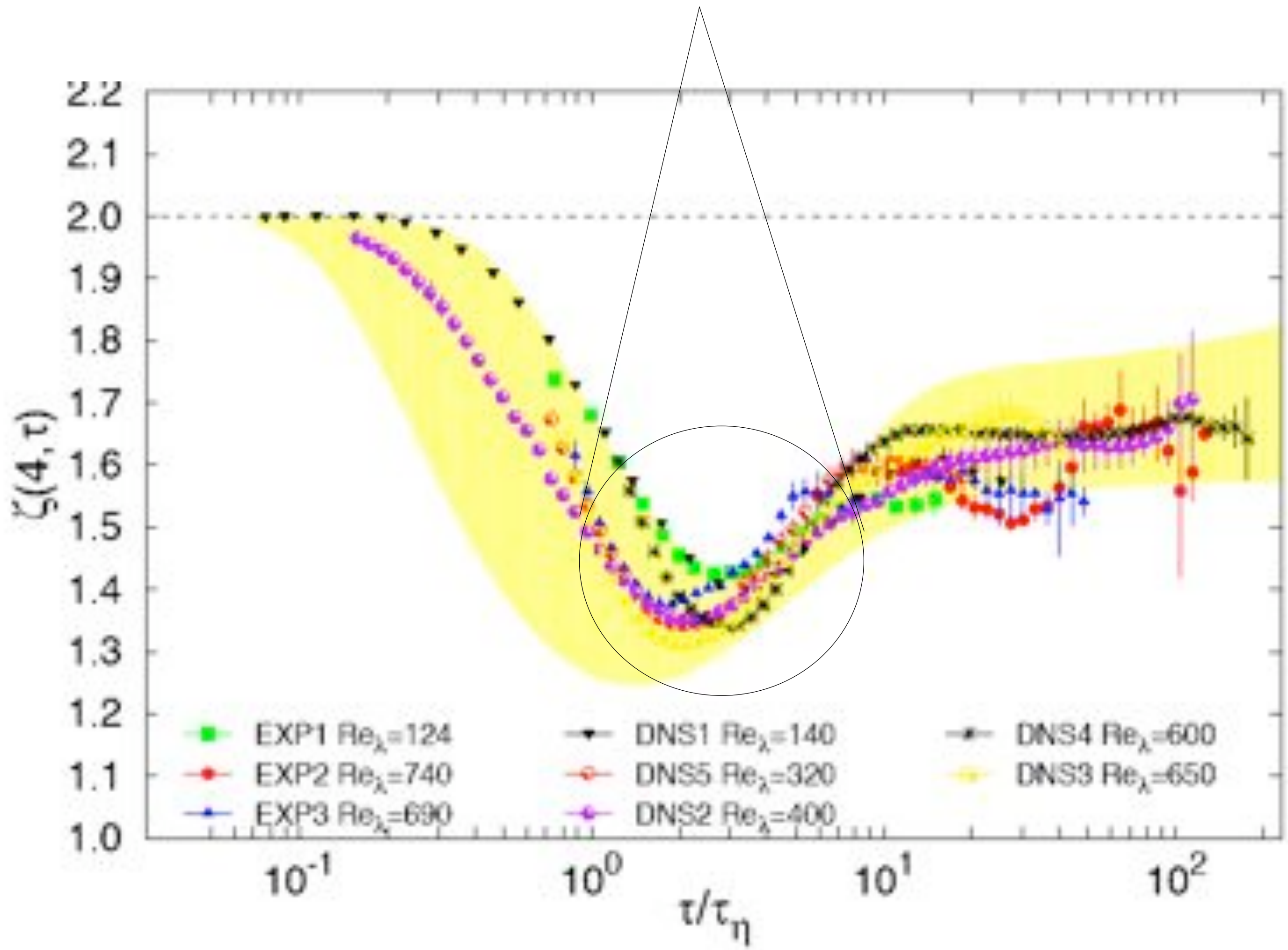
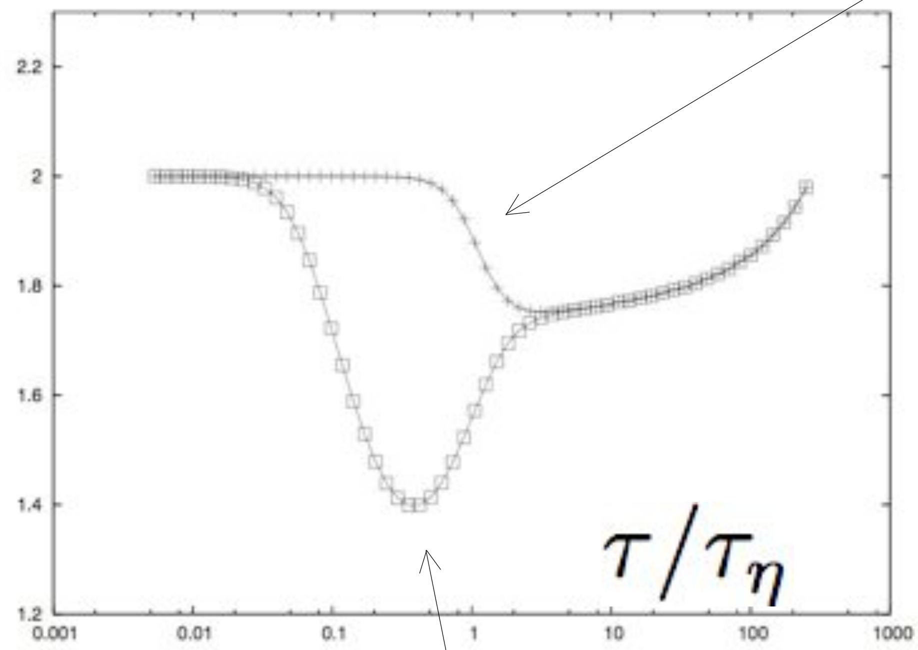


FIG. 1: Log-Lin plot of the local exponent for the fourth moment, $\zeta(4, \tau)$, averaged over the three velocity components, as a function of the normalised time lag τ/τ_η. Data sets come from three experiments (EXP) (see table 1) and five direct numerical simulations (DNS) (see table 2). Error bars are estimated out of the spread between the three components, but for EXP1 and EXP3 where only two components have been considered because of large systematic anisotropic effects in the third one. Each data set is plotted only in the time range where the known experimental/numerical limitations are certainly not affecting the

MultiFractal <u>WITHOUT</u> DISSIPATIVE FLUCTUATING $\tau_\eta(h)$

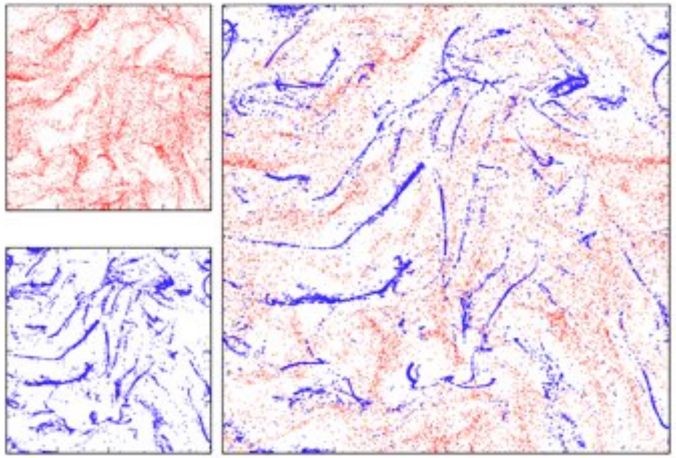
$$\zeta_p(\tau) = \frac{\mathrm{d}\log(S_p(\tau))}{\mathrm{d}\log(S_2(\tau))}$$



MultiFractal <u>WITH</u> DISSIPATIVE FLUCTUATING $\tau_\eta(h)$

DNS

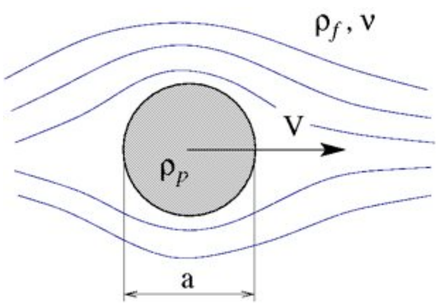
heavy



light

+s and -s

•EQUATION OF MOTION AND ASSUMPTIONS



$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

$$\frac{d\boldsymbol{V}}{dt} = \beta \frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t)-\boldsymbol{V}}{\tau}$$

β<1 heavy particles

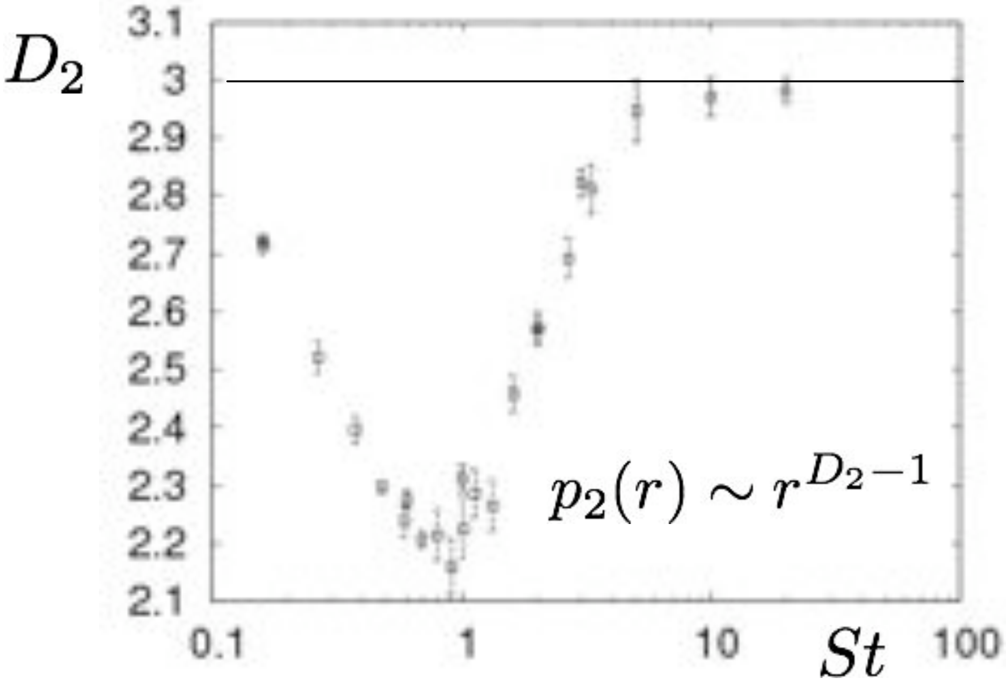
β>1 light particles

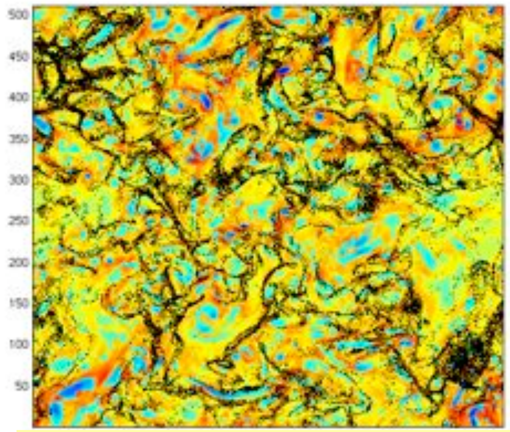
$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

Drag: **Stokes Time**

$$\tau = \frac{a^2}{3\nu\beta}$$

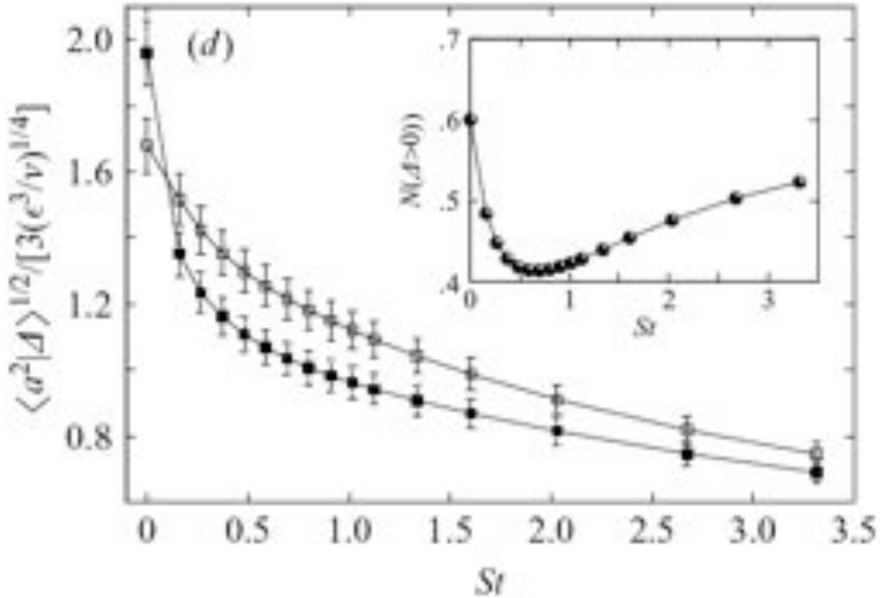
Preferential Concentration





■ Rotating regions Δ > 0
□ Strain regions Δ < 0

$$\Delta = \left(\frac{\det[\hat{\sigma}]}{2}\right)^2 - \left(\frac{\mathrm{Tr}[\hat{\sigma}^2]}{6}\right)^3 \quad \Delta \le 0 \quad \Delta > 0$$

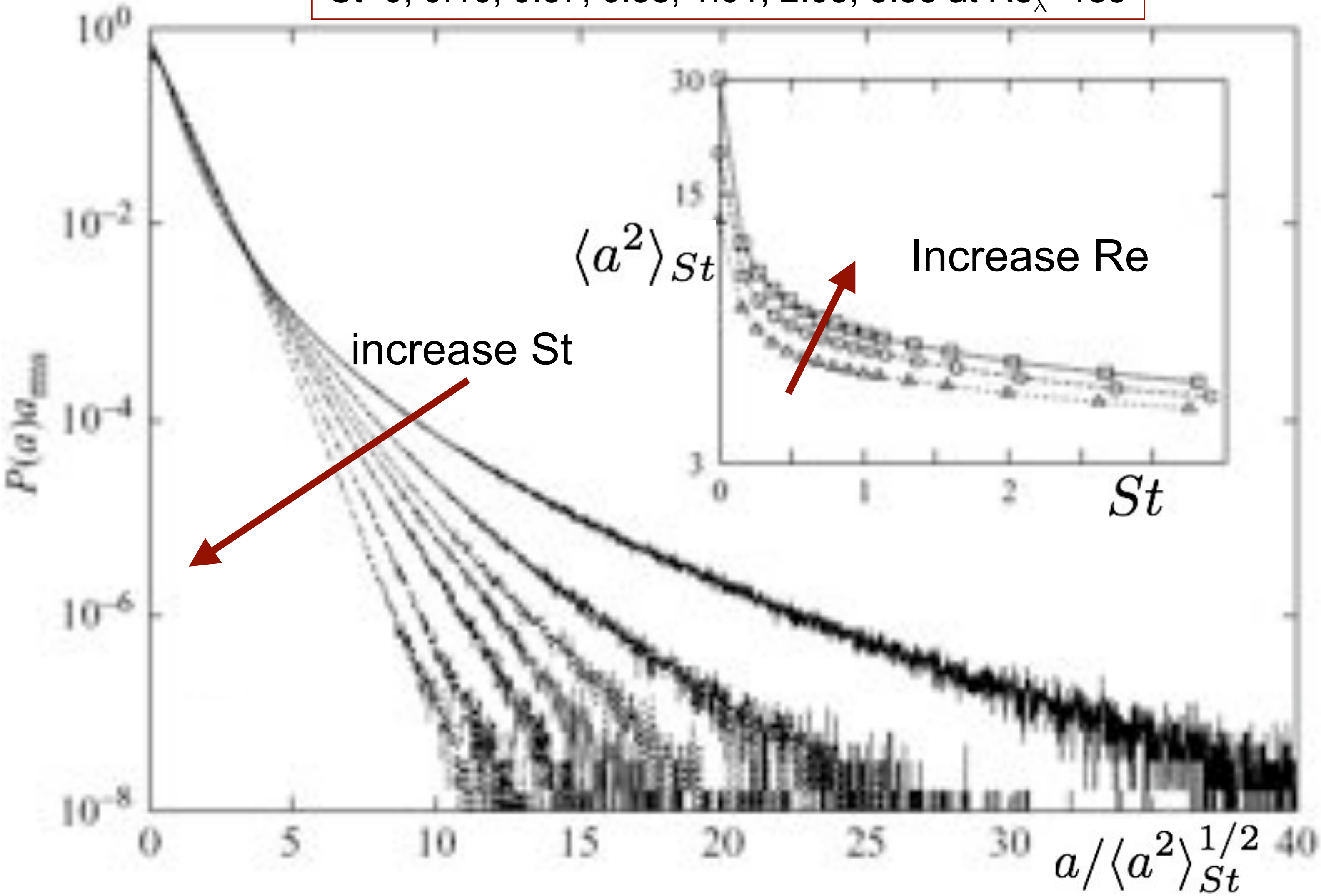


Okubo–Weiss parameter Q
Is the determinant of the strain matrix:

Q: universalita'?

Acceleration: pdf(a) vs. St

St=0, 0.16, 0.37, 0.58, 1.01, 2.03, 3.33 at Re_λ=185



Q: how to include inertia in Multifractal phenomenology?

J. Bec, L.B., G. Boffetta, M. Cencini, A. Lanotte, S. Musacchio & F. Toschi JFM 550, p 349, 2006

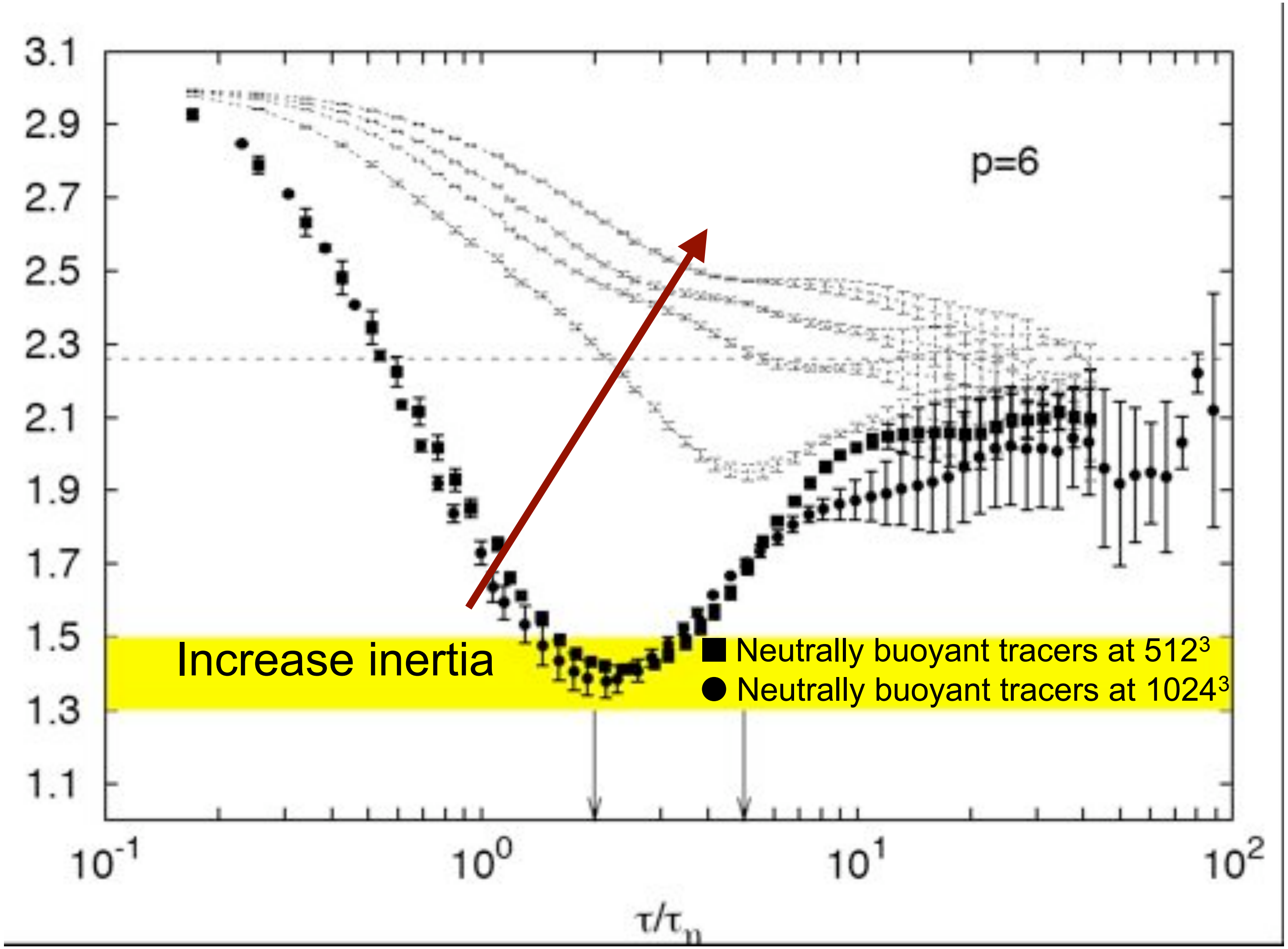
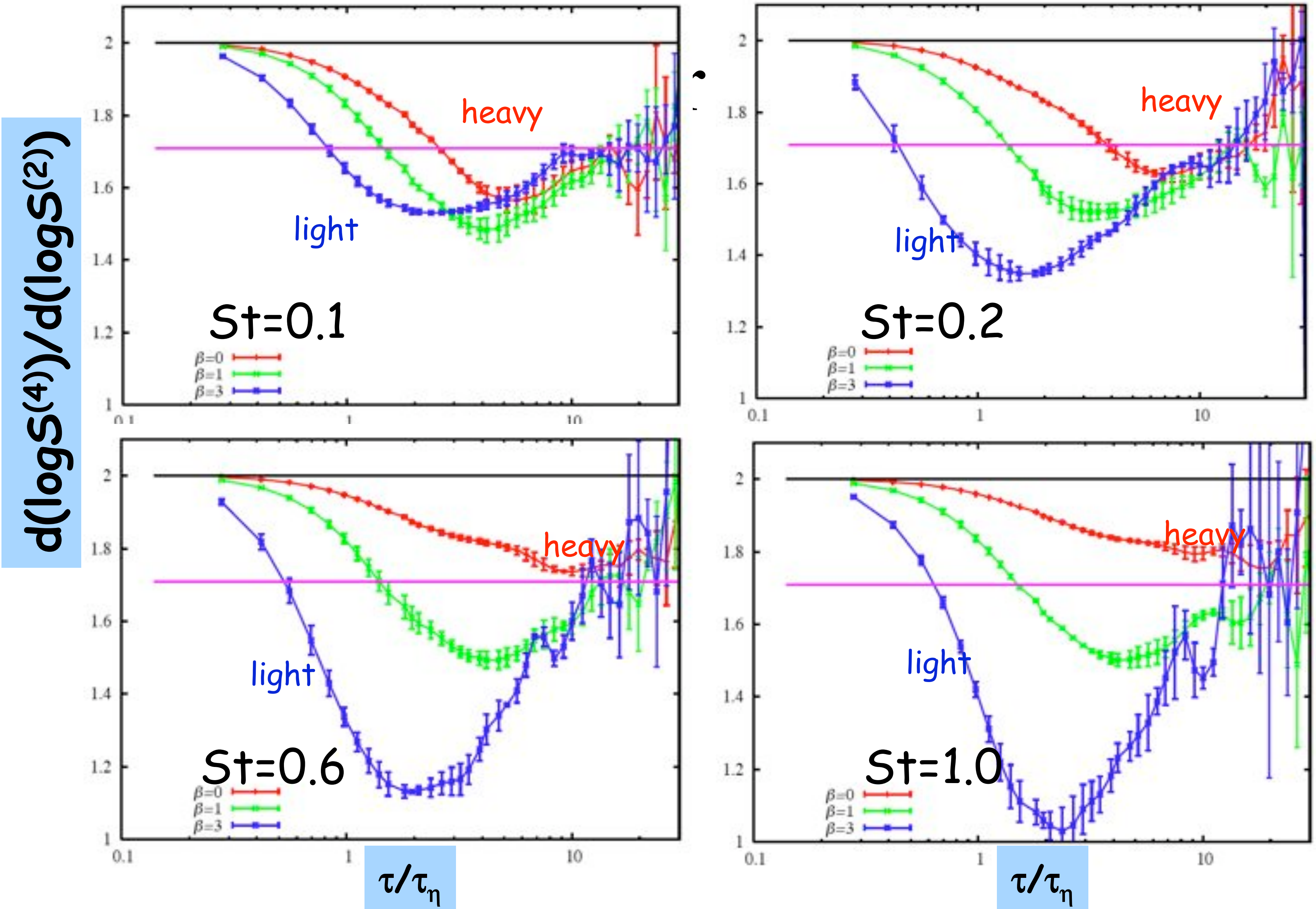


Figure from: On the effects of vortex trapping on the velocity statistics of tracers and heavy particle in turbulent flows
J. Bec, L. B., M. Cencini, A. S. Lanotte, and F. Toschi, PoF 18, 081702, 2006.


d(logS^(4))/d(logS^(2))
heavy
light
St=0.1
St=0.2
St=0.6
St=1.0
β=0
β=1
β=3
τ/τη

CLOSED

- Lagrangian Structure Functions are Intermittent.

- Intermittency increases considerably around dissipative scales.

- Lagrangian Structure Functions are UNIVERSAL.

- Lagrangian anisotropy decays (how fast? Need more careful checks with other flow configurations).

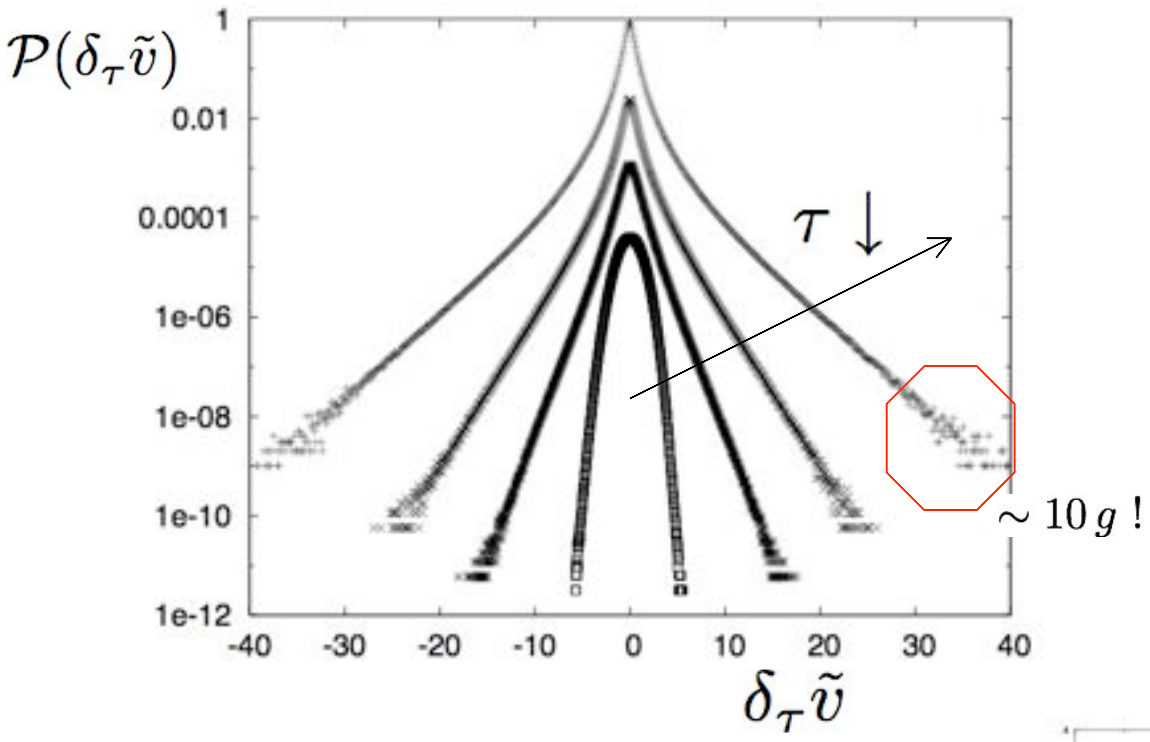
- Translation from Eulerian to Lagrangian Multifractal works well: giving a prediction with only 1 important free parameter connected to the transition between inertial and viscous ranges.

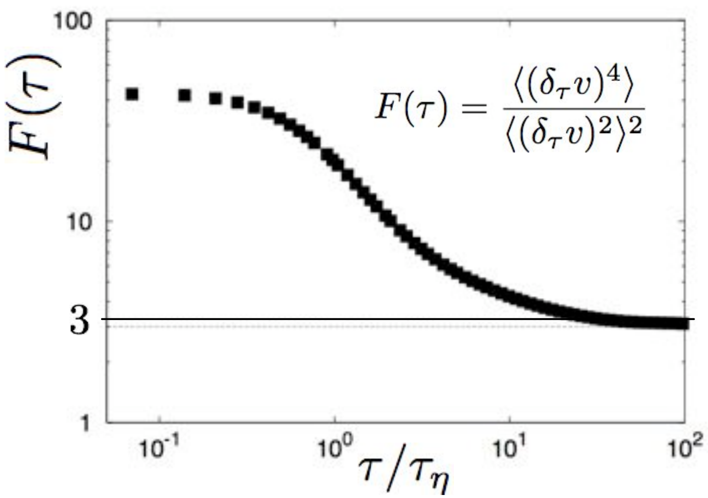
OPEN

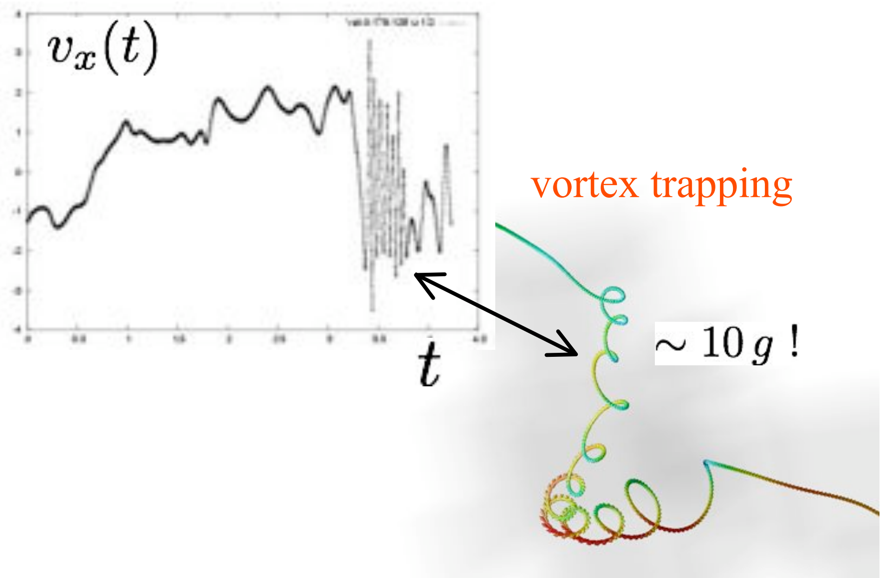
- Can we have a MF/LDT also for heavy and light particle?

We don’t know. It looks difficult -> we need to include preferential concentration in to the stochastic model.

vortex filaments: dog or tail?







- Kraichnan et al: superposition of random vortex filaments: k41 scaling with longitudinal=transverse scaling.
- Belin, Maurer, Tabeling & Willaime: filaments transition (statistical instability) at Re ~ 700
- Chorin: collection of sel-avoiding vortex filaments -> fractal structure
- Passot Politano et al: influence of vortex filaments on the energy spectrum
- Migdal: loop turbulence, statistics driven by velocity circulation

8.9.2 Statistical signature of vortex filaments: dog or tail?

Having identified 'simple' geometric objects, the vortex filaments, in turbulent flows, it is natural to ask if any of the known statistical properties of turbulence can be thus explained. Are the vortex filaments the *dog* or the *tail*? In the former case, they would be essential to explain the energetics and the scaling properties of high-Reynolds-number flow. In the latter case, they would have only marginal signatures, for example on the tails of p.d.f.s of various small-scale quantities and on the exponents ζ_p for large *p*s.

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TOSCHI, F	58	49.1525 %
BENZI, R	39	33.0508 %
SBRAGAGLIA, M	19	16.1017 %
SUCCI, S	18	15.2542 %
CENCINI, M	16	13.5593 %
VULPIANI, A	14	11.8644 %
LANOTTE, A	13	11.0169 %
BOFFETTA, G	12	10.1695 %
CELANI, A	12	10.1695 %
LANOTTE, AS	12	10.1695 %
TRIPICCIONE, R	8	6.7797 %
VERGASSOLA, M	8	6.7797 %
BEC, J	7	5.9322 %
CHIBBARO, S	6	5.0847 %
DIOTALLEVI, F	6	5.0847 %
VERGNI, D	6	5.0847 %
PROCACCIA, I	5	4.2373 %

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